Laplace Transform Properties and Brief Table

Definitions

- Function $f(t)$ is *piecewise continuous* on the finite interval $[a,b]$ if it has only finitely many discontinuities c in the interior of $[a,b]$, at which points both $\lim_{t\to c^-} f(t)$ and $\lim_{t\to c^+} f(t)$ exist and are finite, so $f(t)$ has a *jump discontinuity* at c. The function $f(t)$ is piecewise continuous on an infinite interval if it is piecewise continuous on every finite subinterval.

- Function $f(t)$ is of *exponential order* r if there exist positive constants T, M such that $|f(t)| \leq Me^{rt}$ for $t \geq T$.

- If $f(t)$ is piecewise continuous on $[0,\infty)$ and of exponential order r, then the Laplace transform $\mathcal{L}\{f(t)\}(s) = F(s)$ of $f(t)$ is defined as a function of s by the formula

$$F(s) = \int_0^\infty e^{-st} f(t)dt.$$

  Thus $F(s)$ is defined for all s for which this improper integal is finite (this includes any $s > r$.)

- If $F(s) = \mathcal{L}\{f(t)\}(s)$, then we also write $f(t) = \mathcal{L}^{-1}\{F(s)\}$and call $f(t)$ **the** *inverse Laplace transform* of $F(s)$, a terminology that is justified by the fact that any two piecewise continuous functions with the same Laplace transform may differ only at their points of discontinuity.

In the following domain of $f(t)$ is $t \geq 0$ and $F(s)$ is $s > 0$ unless otherwise stated, n is a non-negative integer. The third column consists of Laplace transforms of entries in the second column, and $u(t)$ is the Heaviside (unit) step function which is zero for $t < 0$ and one otherwise.

	$\mathbf{f(t)}$	$\mathbf{F(s) = L\{f\}(s)}$
1.	$cf(t) + dg(t)$	$c\mathcal{L}\{f\}(s) + d\mathcal{L}\{g\}(s)$
2.	$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots$ $\cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
3.	$e^{at}f(t)$	$F(s-a),\ s > \max\{a,0\}$
4.	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
5.	1	$\frac{1}{s}$
6.	t^n	$\frac{n!}{s^{n+1}}$
7.	$\sin(bt)$	$\frac{b}{s^2+b^2}$
8.	$\cos(bt)$	$\frac{s}{s^2+b^2}$
9.	$\sinh(bt)$	$\frac{b}{s^2-b^2}$
10.	$\cosh(bt)$	$\frac{s}{s^2-b^2}$
11.	$f(t-a)u(t-a),\ a > 0$	$e^{-as}F(s)$
12.	$f(t)u(t-a),\ a > 0$	$e^{-as}\mathcal{L}\{f(t+a)\}(s)$
13.	$f(t) = f(t+T),\ T > 0$	$\int_0^T e^{-st}f(t)dt/\left(1-e^{-sT}\right)$
14.	$t^r,\ r > -1$	$\frac{\Gamma(r+1)}{s^{r+1}},\ \Gamma(x) = \int_0^\infty e^{-u}u^{x-1}du$
15.	$(f \star g)(t) = \int_0^t f(t-v)g(v)dv$	$F(s)G(s)$
16.	$\frac{f(t)}{t},\ \lim_{t\to 0^+}\frac{f(t)}{t}$ defined.	$\int_s^\infty F(\sigma)\,d\sigma$