Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, but clearly so indicate. You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., \( \pi \)) are preferred to inexact (e.g., 3.14). Make all obvious simplifications, e.g., 0 rather than \( \sin \pi \). Point values of problems are given in parentheses. Notes or text in any form not allowed. The only electronic equipment allowed is a calculator. Table entries for Laplace transforms may be used freely, unless otherwise stated. However, you must show enough detail so that it is clear that you are using table entries.

(10) 1. Classify each of the following systems as linear (L), nonlinear (NL), autonomous (A) or non-autonomous (NA).
(a) \( 3y^2 y' + y^3 = e^x \)
(b) \( \frac{dy}{dt} + \sin(y) = 0 \)
(c) \( \frac{dx}{dt} = y, \frac{dy}{dt} = x + y \)
(d) \( \frac{d^2y}{dt^2} + \frac{dy}{dt} = t^2y \)

(14) 2. Solve these DEs and IVPs
(a) \( (1 + x^2) \frac{dy}{dx} = 2y \)
(b) \( \frac{1}{y} \frac{dy}{dx} = y^3(2x + 1), \quad y(0) = 1. \)
(c) \( \frac{dy}{dt} - \frac{2}{t} y = t^2 e^t. \)

(12) 3. Use Euler’s Method with a stepsize of \( h = 0.5 \) to compute the approximate solution on the interval [0,1] to the IVP
\( \frac{dy}{dx} = 1 + xy^2, \quad y(0) = 1. \)

(10) 4. Sketch the phase line, classify equilibria and sketch representative solutions for the equation \( \frac{dy}{dx} = y(y^2 - 1). \)

(12) 5. Find the general solution to this damped oscillator problem: \( 2y'' + 4y' + 2y = 0. \)

(14) 6. Compute the equilibria and trajectories of the system \( \frac{dx}{dt} = (1 - y), \frac{dy}{dt} = xy. \)

(15) 7. Given that the linear system \( Y' = AY \) with constant coefficient matrix \( A \) has eigenvalues \(-1, -2\), with corresponding eigenvectors \( \begin{bmatrix} 3 \\ 2 \end{bmatrix} \) and \( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \), respectively, write out the straight line solutions and the general solution to the system. Use this to find the solution to the system that satisfies \( Y(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \).

(15) 8. Use the eigenvalue method to find the general solution to the linear system \( x' = x + y, \quad y' = -x + y \). (You
(6) 9. A linear system \( Y' = AY \) has repeated eigenvectors \( 2, 2 \), eigenvector \( \begin{bmatrix} 3 \\ 1 \end{bmatrix} \) and generalized eigenvector \( \begin{bmatrix} -1 \\ 1 \end{bmatrix} \). Use these facts to find a general solution to the system and write out the form of each component of this solution.

(24) 10. Given the IVP \( y'' + 3y' + 2y = t, \ y(0) = 0, \) and \( y'(0) = 0. \)
(a) Find the general solution to the differential equation by the method of undetermined coefficients.

(b) Find the solution to the IVP by using Laplace transforms.

(14) 11. In each case below, find the form of a particular solution to the differential equation. (Do NOT explicitly determine the unknown coefficients.)
(a) \( (D(D - 2)(D - 1))[y] = t + e^{3t} \)

(b) \( y'' + 5y' + 6y = \cos(4t) \)

(14) 12. Find general solutions to the following equations
(a) \( y'' - 4y' + 4y = 0 \)

(b) \( y''' + y' = 0 \)

(12) 13. You are given that the function \( f(t) \) is defined by \( f(t) = \begin{cases} 0, & \text{if } t < 1, \\ t, & \text{if } 1 \leq t < 4, \\ 0, & \text{if } 4 \leq t. \end{cases} \)

(a) Compute \( L \{ f(t) \} \) directly from the definition of Laplace transform.

(b) Express \( f(t) \) in terms of step functions and use the tables to compute \( L \{ f(t) \} \).

(18) 14. Find the inverse Laplace transforms.
(a) \( Y(s) = \frac{1}{s(s^2 + 4)} + \frac{1}{s^4}, \ L^{-1} \{ Y(s) \} = \)

(b) \( Y(s) = \frac{s + 3}{(s - 1)^2 + 4}, \ L^{-1} \{ Y(s) \} = \)

(c) \( Y(s) = \frac{1}{(s - 1)(s + 1)(s + 2)}, \ L^{-1} \{ Y(s) \} = \)

(10) 15. Find the Laplace transforms.
(a) \( L \{ e^{-3t} \cos 2t \} = \)

(b) \( L \left\{ \int_0^t e^{3v} (t - v) \, dv \right\} = \)