(Note: This sample exam is about 1.5 questions too long.)

(18) 1. Solve the IVP \[ \frac{dx_1}{dt} = -2x_1 + x_2, \quad \frac{dx_2}{dt} = 4x_1 - 2x_2, \quad x_1(0) = 1, \quad x_2(0) = 0 \] by the method of eigenvalues. (Arithmetic check: eigenvalues are 0, -4.)

(18) 2. Find the general solution to \[ \frac{dx_1}{dt} = -2x_1 + x_2, \quad \frac{dx_2}{dt} = -1x_1 - 4x_2, \] by the method of eigenvalues. (You may assume that -3, -3 are the eigenvalues of this system and \([1, -1]^T\) is an eigenvector.)

(10) 3. Sketch typical phase planes for a linear system \( \frac{dx}{dt} = Ax + By, \quad \frac{dy}{dt} = Cx + Dy \) which has at the origin a
(a) stable node. \hspace{1cm} (b) saddle. \hspace{1cm} (c) center.

(10) 4. Solve the DE \( \frac{dx}{dt} = 2y, \quad \frac{dy}{dt} = -4x, \) sketch a few representative solutions and classify the critical point at the origin.

(12) 5. Find the inverse Laplace transforms.
(a) \( Y(s) = \frac{1}{(s + 9)s^2}. \quad L^{-1} \{ Y \} = \)
(b) \( Y(s) = \frac{s}{s^2 - 2s + 5} + \frac{1}{s^4}. \quad L^{-1} \{ Y \} = \)

(12) 6. Find the Laplace transforms.
(a) \( L \left\{ \frac{\sinh t}{t} \right\} = \)
(b) \( L \left\{ e^{3t^2} + t^3 \cos(2t) \right\} = \)

(12) 7. Express \( f(t) = \int_0^t e^v \sin(t - v)dv \) in terms of convolutions. Then use this expression to find the Laplace transform of \( f(t). \)

(18) 8. Solve the IVP \( y'' - 4y = e^{2t}, \quad y(0) = 0, \quad y'(0) = 0 \) by the method of Laplace transforms.