Quiz 5 **Math 208** **Fall 2009**

Name:______________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14), and you should make obvious simplifications. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is optional.

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(3) **1.** (Exer. 14.3.3) Is the vector field $\mathbf{F} = y\mathbf{i} + (x+z)\,\mathbf{j} - y\mathbf{k}$ conservative?

SOLUTION. No, for if $\mathbf{F} = \langle M, N, P \rangle$, we must have $\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$ in a conservative vector field, yet $\frac{\partial(x+z)}{z} = 1$ and $\frac{\partial(-y)}{y} = -1$ in our case.

(6) **2.** (Handout Exer. 4) Find a potential function for the vector field $\mathbf{F}(x,y) = \langle x-5, 3y^2+7 \rangle$.

SOLUTION. This one's easy, since if we assume $\mathbf{F} = \nabla f = \langle f_x, f_y \rangle$, then $f_x = x - 5$, so

$$\begin{aligned} f &= \int f_x dx = \frac{x^2}{2} - 5x + C(y) \\ f_y &= C'(y) = 3y^2 + 7 \\ C &= \int C'(y)\,dy = 3\frac{y^3}{3} + 7y \\ f &= \frac{x^2}{2} - 5x + y^3 + 7y. \end{aligned}$$

(3) **3**. State the flow (circulation) form of Green's theorem for the vector field $\mathbf{F} = \langle M, N \rangle$ using either the abstract or concrete formulation.

SOLUTION. If simple closed curve C is oriented positively with respect to its interior R, and $\mathbf{F}(x,y)$ is a smooth vector field defined on C and R then

Abstract:

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C \mathbf{F} \cdot \mathbf{T} ds = \iint_R \nabla \times \mathbf{F} \cdot \mathbf{k} dA.$$

Concrete:

$$\oint_C M\,dx + N\,dy = \iint_R (N_x - M_y)\,dA.$$

(6) **4.** (Exer. 14.4.22) Let $\mathbf{F} = \frac{1}{2}\langle x, y \rangle$ and let R be the interior of the ellipse C parametrized by $\mathbf{r}(t) = \langle a\cos t, b\sin t \rangle$, $0 \le t \le 2\pi$. Use the flux form of Green's theorem to calculate the area of the ellipse.

SOLUTION. The flux form says

$$\oint_C \mathbf{F} \cdot \mathbf{n} ds = \oint_C M\,dy - N\,dx = \iint_R (M_x + N_y)\,dA,$$

and since $M = \frac{1}{2}x$, $N = \frac{1}{2}y$, we have

$$\frac{1}{2}\oint_C x\,dy - y\,dx = \iint_R \left(\frac{1}{2} + \frac{1}{2}\right) dA = \iint_R dA.$$

Now according to the parametrization, $x = a\cos t$, $dx = -a\sin t$, $y = b\sin t$, $dy = b\cos t$, so that

$$\frac{1}{2}\oint_C x\,dy - y\,dx = \frac{1}{2}\int_0^{2\pi} ((a\cos t)(b\cos t) - b\sin t(-a\cos t))\,dt = \frac{ab}{2}\int_0^{2\pi} \left(\cos^2 t + \sin^2 t\right) dt = \pi ab.$$