(6) 1. (Exercise 13.5.3) Write two different iterated integrals for the volume of the tetrahedron cut off in the first octant by the plane $6x + 3y + 2z = 6$. Sketch the solid.

**Solution.**

From the graph we see that

$$\iiint_D 1 \cdot dV = \int_0^1 \int_0^{-\frac{1}{2}y} \int_0^{3-3x-\frac{3}{2}y} dz \, dy \, dx$$

$$= \int_0^2 \int_0^{-\frac{1}{2}y} \int_0^{3-3x-\frac{3}{2}y} dz \, dx$$

$$= \int_0^3 \int_0^{2-\frac{3}{2}z} \int_0^{1-\frac{1}{2}y-\frac{3}{2}z} 1 \, dx \, dy \, dz.$$  

(6) 2. (Exercise 13.7.15) $D$ is the solid inside the right circular cylinder whose base is the circle $r = 2 \sin \theta$ in the $xy$-plane and top is given by $z = 4 - y$. Set up an iterated integral for $\iiint_D f(x, y, z) \, dV$ in cylindrical coordinates in the order $dz \, r \, dr \, d\theta$. Sketch $D$.

**Solution.**

Cylinder $r = 2 \sin \theta$ is the same as $r^2 = 2r \sin \theta$, that is, $x^2 + y^2 = 2y$, $x^2 + (y - 1)^2 = 1$, a circle of radius 1, center at $(0, 1)$. The bottom of the solid is $z = 0$ and the top is the plane $z = 4 - y$. Thus $\iiint f(x, y, z) \, dV$ is the iterated integral

$$\int_0^{4 \pi} \int_0^{\pi/2} \int_0^{4 - r \sin \theta} f(r \cos \theta, r \sin \theta, z) \ r \ dz \ dr \ d\theta.$$  

(8) 3. (Exercise 5, Handout) Express the mass of an object inside the sphere $x^2 + y^2 + z^2 = 4z$ and below the cone $z = \sqrt{3x^2 + 3y^2}$ as an iterated integral in spherical coordinates, if the density is given by $\delta(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$. Sketch the solid.

**Solution.**

Complete the square to see the sphere is given by $x^2 + y^2 + (z - 2)^2 = 4$, so has radius 2, center at $(0, 0, 2)$. In spherical coordinates it is

$$\rho^2 = 4 \rho \cos \phi$$

or $\rho = 4 \cos \phi$. Cone angle from the vertical is the same as the line $z = \sqrt{3y}$ with the $z$-axis. Take $y = 1$, get $z = \sqrt{3}$. 30-60-90 right triangle as in the graph, so angle $\phi$ starts at $\phi = \pi/6$. Hence the mass is $M =$

$$\iiint_D \delta(x, y, z) \, dV = \int_0^{\pi/2} \int_0^{\pi/6} \int_0^{4 \cos \phi} \frac{1}{\rho^2 \sin \phi} \ d\rho \ d\phi \ d\theta$$

$$= \int_0^{\pi/2} \int_0^{\pi/6} \int_0^{4 \cos \phi} \sin \phi \ d\rho \ d\phi \ d\theta.$$