Quiz 2 Key **Math 208** **Fall 2009**

Name:_____ **Score:**_____

Instructions: Show your work in the spaces provided below for full credit. You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14), and you should make obvious simplifications. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is optional.

(6) **1.** (Exer. 12.1.6) Find the domain and range of the function $f(x,y) = y/x^2$. Determine if the domain is open, closed or bounded and describe the level curves of this function.

SOLUTION. The domain is

$$\{(x,y) \mid x \neq 0\},$$

that is, the entire plane minus the y-axis. This set is open and unbounded.

Range is the set off all reals.

Level curves are of form $c = y/x^2$ or $y = cx^2$ with $x \neq 0$, i.e., parabola or x-axis, punctured at $x = 0$.

(6) **2.** (Exer. 12.3.44) Find all second-order partials of the function $h(x,y) = xe^y + y + 1$.

SOLUTION. Direct calculation gives

$$\begin{aligned} h_x &= e^y \\ h_{xx} &= 0 \\ h_{xy} &= e^y = h_{yx} \\ h_y &= xe^y + 1 \\ h_{yy} &= xe^y \end{aligned}$$

(8) **3.** (Exer. 1, Handout) Find w_t in terms of x, y, s and t, if $x(s,t) = s\cos(2t)$, $y(s,t) = 2t - s$, and $w(s,t) = f(x(s,t), y(s,t))$ with $f(x,y) = x\tan^{-1}(y)$. (Set up a chain rule formula first.)

SOLUTION. Note that w is dependent, x, y are intermediate and s, t are independent. Thus we have the chain rule

$$\begin{aligned} w_t &= \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} \\ &= \frac{\partial\left(x\tan^{-1}(y)\right)}{\partial x}\frac{\partial\left(s\cos(2t)\right)}{\partial t} + \frac{\partial\left(x\tan^{-1}(y)\right)}{\partial y}\frac{\partial(2t-s)}{\partial t} \\ &= \tan^{-1}(y)\left(-2s\sin(2t)\right) + \frac{x}{1+y^2}2 \\ &= -2s\tan^{-1}(y)\sin(2t) + \frac{2x}{1+y^2} \end{aligned}$$