Quiz 1 Key **Math 208** **Fall 2009**

Name:__________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is optional.

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(10) **1.** (Exer. 10.4.18) Given points $P(-2,2,0)$, $Q(0,1,-1)$, and $R(-1,2,-2)$.
(a) Find the area of the triangle with vertices P, Q, and R.
(b) Find a unit vector perpendicular to the plane PQR.
(c) Find an equation for the plane PQR.

SOLUTION. (a) Let $\mathbf{a} = \overrightarrow{PQ} = \langle 0 - -2, 1-2, -1-0\rangle = \langle 2,-1,-1\rangle$, $\mathbf{b} = \overrightarrow{PR} = \langle -1 - -2, 2-2, -2-0\rangle = \langle 1,0,-2\rangle$ and calculate

$$\mathbf{a}\times\mathbf{b} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & -1 \\ 1 & 0 & -2 \end{bmatrix} = \langle (-1)(-2) - 0(-1), -(2(-2) - (-1)1), 2\cdot 0 - (-1)1\rangle = \langle 2,3,1\rangle .$$

Then the area is half the parallelogram determined by $\mathbf{a}, \mathbf{b}$, i.e.,

$$\frac{1}{2}|\mathbf{a}\times\mathbf{b}| = \frac{1}{2}\sqrt{4+9+1} = \frac{1}{2}\sqrt{14}$$

.

(b) $\mathbf{a}\times\mathbf{b}$ is perpendicular to the plane, so a unit vector is

$$\frac{1}{|\mathbf{a}\times\mathbf{b}|}\mathbf{a}\times\mathbf{b} = \frac{1}{\sqrt{14}}\langle 2,3,1\rangle$$

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(c) Use point P and $\mathbf{a}\times\mathbf{b}$ to obtain $0 = 2(x - -2) + 3(y-2) + 1(z-0)$ or

$$2x + 3y + z = 2$$

.

(5) **2.** (Exer. 10.5.6) Find parametric equations for the line through $(3,-2,1)$ and parallel to the line $x = 1+2t$, $y = 2-t$, $z = 3t$.

SOLUTION. Since $\langle 2,-1,3\rangle$ is parallel to the given line , and $(3,-2,1)$ is on the line, the equations are $x = 3+2t$, $y = -2-t$, $z = 1+3t$, i.e.,

$$\begin{aligned} x &= 3+2t \\ y &= -2-t \\ z &= 1+3t. \end{aligned}$$

(5) **3.** (Exer. 10.6.35) Identify this surface and its traces (do not sketch): $y = -(x^2+z^2)$.

SOLUTION. The surface is an elliptic paraboloid.

The trace with $x=0$ gives parabola $y = -z^2$.

The trace with $y=0$ gives point (degenerate circle) $x^2+z^2=0$.

The trace with $z=0$ gives parabola $y=-x^2$.