Standard surfaces in spherical coordinates

As in cylindrical coordinates, be aware that replacing θ everywhere by $\theta - \theta_0$ in any of these equations simply rotates the surface counterclockwise (as viewed from above) around the z-axis by θ_0. Also, replacing (ρ,ϕ,θ) by $(\rho,-\phi,\theta+\pi)$, $(\rho,\phi+2n\pi,\theta)$ or both one after the other can change things to where $0 \le \phi \le \pi$, as required for spherical integrals. Finally, if $\rho < 0$, replacing (ρ,ϕ,θ) by $(-\rho,\pi-\phi,\theta+\pi)$ will change an equation where ρ is negative at a point to one where ρ is positive at that point (while keeping $0 \le \phi \le \pi$ if it was there before), or vice versa. Using $\rho < 0$ is never needed in spherical integrals, and usually leads to the incorrect region or covering part of the region twice.

In the following table, c is a constants with the indicated restrictions.

Equation	Shape
$\rho = c,\ c > 0$	Sphere of radius c centered around the origin.
$\phi = c,\ 0 \le c \le \pi$, assuming $\rho \ge 0$	If $c = 0$, this is the positive z-axis, not a surface. If $0 < c < \frac{\pi}{2}$, this is $r = z\tan(c)$ or $z = r\cot(c)$, a cone going up from the origin around the positive z-axis. If $c = \frac{\pi}{2}$, this is $z = 0$, the xy-plane. If $\frac{\pi}{2} < c < \pi$, this is again $r = z\tan(c)$ or $z = r\cot(c)$, a cone going up from the origin around the positive z-axis. If $c = \pi$, this is the negative z-axis, again not a surface.
$\theta = c$	Half-plane starting at the z-axis (same as in cylindrical coordinates).
Any equation involving only ρ and ϕ, not involving θ	Graph this in the rz-plane – the variables ρ and ϕ form a polar coordinate system for that plane, but with the z-axis acting in place of the x-axis and the r-axis acting as the y-axis – and the surface is what you get by rotating that curve around the z-axis. Thus e.g. $\rho = 4\cos(\phi)$ in the rz-plane is a circle of radius 2 sitting on the r-axis at the origin, and when rotated becomes a sphere of radius 2 sitting on the xy- plane at the origin.

If you see a surface that doesn't fit in one of these categories, try to write the equation as $\rho = f(\phi,\theta)$ and proceed from there, keeping in mind you almost always want to integrate in the order $d\rho\, d\phi\, d\theta$. But visualizing the graph of the surface will probably be easier thinking of the equation in rectangular or cylindrical coordinates.