Math 208
Cylindrical and spherical coordinates problems

Set up and evaluate each of the following in either cylindrical or spherical coordinates, whichever is more appropriate:

1. $\int_Q x\,dV$, where Q is the region with $x \geq 0$, inside the sphere $x^2 + y^2 + z^2 = 16$, and below the cone $z = \sqrt{x^2 + y^2}$.

2. $\int_0^2 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} \int_{-2\sqrt{x^2+y^2}}^{2\sqrt{x^2+y^2}} \frac{x}{x^2+y^2}\,dz\,dy\,dx$

3. $\int_0^2 \int_0^{\sqrt{8-2y^2}} \int_y^{\sqrt{8-y^2-z^2}} \frac{1}{\sqrt{x^2+y^2+z^2}}\,dx\,dz\,dy$

4. $\int_{-2}^0 \int_{2y^2}^{-4y} \int_{-y}^{\sqrt{z-y^2}} \frac{1}{x^2+y^2}\,dx\,dz\,dy$

5. Find the mass and *z*-coordinate of the center of mass of the object inside the sphere $x^2 + y^2 + z^2 = 4z$ and below the cone $z = \sqrt{3x^2 + 3y^2}$, if the density is $\delta(x, y, z) = \frac{1}{x^2+y^2+z^2}$.

Answers: 1) $48\pi + 32$ 2) $\frac{32}{3} = 10\frac{2}{3}$ 3) π

4) $\pi - 2$ 5) mass = 3π, $\overline{z} = 0.75$