Math 208
Gradient applications additional problems

1. Let $f(x,y) = -0.25x^2 - 0.25y^2$.
   a) Find $\nabla f(-3,4)$.
   b) Find an equation for the level curve of f going through the point (−3,4).
   c) Find an equation for the tangent line at (−3,4) to the equation found in (b).
   d) Find a system of parametric equations and a non-parametric equation for the normal line to the level curve of f going through the point (−3,4).
   e) In which direction does f decrease most rapidly at (−3,4), and what is the rate of change of f at (−3,4) in that direction?
   f) What is the directional derivative of f at (−3,4) in the direction from there toward (0,1)?
   g) Find all unit vectors **u** such that $f_{\mathbf{u}}(-3,4) = 0$.
   h) Find the unit vectors in all directions for which the rate of change of f at (−3,4) equals 2.

2. Let $f(x,y,z) = x^3z - 2yz^2 - 2z$. Simplify all numbers in this problem.
   a) Find $\nabla f(2,-1,3)$.
   b) Find the rate of change of f at (2, −1,3) in the direction from there toward (−4,1,6).
   c) Find an equation for the tangent plane to the level surface $f(x,y,z) = 36$ at the point (2, −1,3).
   d) Find parametric equations for the normal line to the level surface $f(x,y,z) = 36$ at the point (2, −1,3).
   e) Find a unit vector in the direction in which f increases most rapidly at (2, −1,3), and the rate of change of f in that direction.

Gradient applications handout answers:

1. a) $< 1.5, -2 >$ b) $-0.25x^2 - 0.25y^2 = -6.25$, or $x^2 + y^2 = 25$
   c) $1.5(x+3) - 2(y-4) = 0$ or equivalent, such as $4y - 3x = 25$
   d) Parametric: $x = -3 + 1.5t$ and $y = 4 - 2t$

   Non-parametric: $\frac{x+3}{1.5} = \frac{y-4}{-2}$ or equivalent such as $4x + 3y = 0$
   e) Direction is direction of $< -1.5, 2 >$, rate of change is -2.5
   f) $\frac{3.5}{\sqrt{2}}$ g) $\mathbf{u} = < 0.8, 0.6 >$ or $\mathbf{u} = < -0.8, -0.6 >$
   h) $< 0, -1 >$ and $< 0.96, -0.28 >$ are the only such unit vectors

2. a) $<36, -18, 18> = 18<2, -1, 1>$ b) $-28\frac{2}{7}$
   c) $2x - y + z = 8$ (or equivalent)
   d) $x = 2 + 2t$, $y = -1 - t$, $z = 3 + t$ (or equivalent)
   e) unit vector $\frac{<2, -1, 1>}{\sqrt{6}}$, rate of change $18\sqrt{6}$