Exam 2 **Math 208** **Fall 2007**

Name:______________________________ **Score:**____________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. The only electronic equipment allowed is a calculator.

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(25) **1.** Let $f(x,y) = x^2 - 4xy + y^3 + 4y$.
(a) Find all critical points of f.

(b) Use the second derivative test to classify the critical points of f.

(c) Sketch the region bounded by $y = x$, $y = 0$ and $x = 2$. Clearly identify all the points at which you should check the value of f in order to find the extrema of f by the EVT. Do NOT actually check them. You may ASSUME that f has no critical points in the interior of the segment $y = x$.

(25) **2.** Let $f(x,y) = 4xy$.
(a) Find the extrema of f subject to the constraint $4x^2 + y^2 = 8$ by the method of Lagrange multipliers.

(b) Find the absolute extrema of f over the region $4x^2 + y^2 \leq 8$.

(15) **3.** Express the volume of the solid above the rectangle $R : 0 \leq x \leq 1$ and $1 \leq y \leq 3$ and bounded by $4 - x - y$ as a double integral and evaluate this integral.

(20) **4.** Evaluate the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \frac{3}{4+y^3}\,dy\,dx$$

by interchanging the order of integration. Clearly sketch the region of integration.

(15) **5.** Convert the iterated integral $\int_{-1}^{0} \int_{\sqrt{3}}^{\sqrt{4-x^2}} y\left(x^2+y^2\right) dy\,dx$ to polar coordinates (do not evaluate it.) Sketch the region of integration for this problem.

(25) **1.**
(a) $\left(\frac{4}{3}, \frac{2}{3}\right)$, $(4, 2)$.
(b) f has a saddle point at $\left(\frac{4}{3}, \frac{2}{3}\right)$ and local minimum at $(4, 2)$.
(c) Interior point $\left(\frac{4}{3}, \frac{2}{3}\right)$, corner points $(0, 0)$, $(2, 0)$, $(2, 2)$ and boundary point $\left(2, 2\sqrt{3}/3\right)$.

(25) **2.**
(a) Maximum of 8 at $(1, 2)$ and $(-1, -2)$. Minimum of -8 at $(-1, 2)$ and $(1, -2)$.

(b) These occur on the boundary points found in (a).

(15) **3.**

$$\iint_R (4 - x - y)\, dA = \int_0^1 \int_1^3 (4 - x - y)\; dy\, dx = 3.$$

(20) **4.**

$$\int_0^1 \int_{\sqrt{x}}^1 \frac{3}{4 + y^3} dy\, dx = \int_0^1 \int_0^{y^2} \frac{3}{4 + y^3}\, dx\, dy = \ln(5) - 2\ln(2)\,.$$

(15) **5.**

$$\int_{-1}^0 \int_{\sqrt{3}}^{\sqrt{4-x^2}} y\left(x^2 + y^2\right) dy\, dx = \int_{\pi/2}^{2\pi/3} \int_{\sqrt{3}\csc(\theta)}^{2} r^4 \sin(\theta)\; dr\, d\theta$$