Linear Algebra Project **Due: Some Date**
Subject: The Art of Proof **Points: 25**

About Project assignments: This assignment should be typed up in some document preparation system such as WordPerfect, MS Word, Latex, etc., or *very* neatly handwritten. Check your grammar and spelling (use a spellchecker if you have one.) Like any good writing exercise, projectlets should have, as Aristotle advises, a beginning (introduction), middle (main body of work) and end (conclusion); these parts can be as small as a paragraph. Remember that part of your grade will be based on the quality of your written work. The paper you turn in should be a mix of equations, formulas and prose. You should write your answers in such a way that it can be read and understood by anyone who knows the material for this course. Your write-up should be self-contained and addressed to classmates, not your instructor. You should cite references and acknowledge all sources in your write-up.

The Problems: We all know that questions about proving or disproving a mathematical assertion can be hard. It would be nice to have some advice about how to approach such problems. This assignment asks you to prove some results and offer some explanation of your own thought processes in arriving at these proofs. Write out your proofs using complete sentences and clear justifications for steps in your arguments. If you need to use other facts about matrices that are in the text, clearly reference them. Conclude with comments that might be helpful to others regarding how you arrived at these proofs.

This set of facts is centered around the set (actually, vector space) $\mathbb{R}^{n\times n}$ of $n\times n$ square matrices with real entries. Think of it as a small essay about symmetric and skew-symmetric matrices.

1. If A and B are matrices of the same size, then
$$(A+B)^T = A^T + B^T.$$
2. If A is any square matrix, then $\frac{1}{2}\left(A+A^T\right)$ is symmetric and $\frac{1}{2}\left(A-A^T\right)$ is skew-symmetric.
3. Every square matrix can be expressed uniquely as the sum of a symmetric matrix and a skew-symmetric matrix.
4. The sets S of all symmetric matrices and K of all skew-symmetric matrices are subspaces of the vector space $\mathbb{R}^{n\times n}$ such that $\mathbb{R}^{n\times n} = S+K$ and $S\bigcap K = \{0\}$.
5. $\dim\mathbb{R}^{n\times n} = \dim S + \dim K$.

If you get stuck, feel free to drop by and see me. This assignment can be completed in two to four pages.