JDEP 384H: Numerical Methods in Business

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Lecture 16, February 13, 2007
110 Kaufmann Center

Outline

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1. BT 3.1: Basics of Numerical Analysis
   - Finite Precision Representation
   - Error Analysis
   - Midterm Review

Floating Point Arithmetic

View the file NumAnalLecture.pdf for a more detailed view of this material.

Finite Precision Arithmetic:

What does it mean?

- It means a finite number of bits of information are used to store an approximation to a given real number.
- Matlab uses IEEE (Institute of Electrical and Electronic Engineers) floating point standards.
- All arithmetic is double precision.
- Thus, eight bytes are used per number, and roughly this gives about $15-16$ decimal digits of precision in a mantissa with exponents roughly in the range $10^{-308}-10^{308}$.

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Floating Point Calculations

We have already seen this in our introduction to Matlab:

```
> eps
> x=1+eps
> y = 10*x
> y-10
> format hex
> 1
> eps
> format
> realmax
> realmin
> realmin/1e15
> realmin/1e16
> 2*realmax
> x = 0/0
> format hex
> x
```

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Rounding Error

Roundoff error:

Common notation for floating point representation of a real number is $\mathrm{fl}(x)$ and The error introduced by mere storage of a real number as floating point is called **roundoff (or rounding) error**.

- In general $\mathrm{fl}(x) \neq x$.
- Error introduced by mere storage of a real number as floating point is called **roundoff (or rounding) error**.
- When we perform an arithmetic operation such as $+$ we don't get exactly what we want, namely $x + y$, but rather

$$\mathrm{fl}(x) \oplus \mathrm{fl}(y)$$

where $\oplus$ is the machine operation.

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Rounding Error

In spite of everything, IEEE standards can give pretty good results.

Try this:

```
> format hex
> y = 0
> x = 1/3
> for ii=1:30, y=y+x; end
> 10-y
```

Now try this with $x = 1/7$, loop to 70.

Then calculate

```
> y = ((2/7+1000)-1000)-2/7
> format
> y
```

Error Terminology

Error Definitions:

Suppose the exact quantity that we want to estimate is x_T and we end up calculating the quantity x_A.

- **Absolute error** of our calculation is

$$\mathrm{Error}(x_A) = |x_A - x_T|$$

- **Relative error** is

$$\mathrm{Rel}(x_A) = \frac{|x_A - x_T|}{|x_T|},$$

provided $x_T \neq 0$.

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Error Terminology

An Application:

Significant digits.

- We say x_A has m significant digits with respect to x_T if m is the largest nonnegative integer for which

$$\mathrm{Rel}(x_A) = \frac{|x_A - x_T|}{|x_T|} \leq 5 \times 10^{-m}.$$

- Use the definition to answer this question: $x_A = 25.489$ has how many significant digits with respect to $x_T = 25.482$.
- Subtraction of nearly equal quantities can cause **catastrophic loss of significance digits**. Addition of them does not cause such a loss.

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Examples

Refer to LectureNotesNumAnal from Example 20 forward to see how this idea is applied to polynomial equations and to linear systems.

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Midterm Review

Rules of the Game:

- The midterm will have two parts.
- An in-class part that is closed book, closed notes, NO calculators, laptops, cell phones, blackberries, etc., etc. This part is worth 100 points. This exam will be administered on Thursday, March 8.
- A take-home part that is worth 35 points. This exam will be available on the class home page Tuesday morning, March 20. It will be due on Friday, March 24, 5:00 pm. It can either be hardcopy (as always, neatly written) or in the form of a single pdf file which you can email to me. You must show all your work, including copies of any scripts that you used and their relevant outputs and acknowledge any sources outside of lecture notes and text.

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Midterm Review

Review Points:

- If it isn't from lecture notes or homework problems, it won't be on the midterm.
- Look at the 2006 midterm. Excise problems 4, 7 and the last part of 8.
- Make sure you can do simple calculations such as some of the board work I have done, matrix, vector and basic statistical calculations.
- Think about conceptual questions like
  - What do convexity and duration have to do with function approximation by derivatives?
  - Explain the terms in the stochastic differential equation

$$dS = \mu S\, dt + \sigma S\, dW\,.$$

  - Explain what the efficient frontier of a portfolio of risky assets with known expected returns is.

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