JDEP 384H: Numerical Methods in Business

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Department of Mathematics

Lecture 11, February 13, 2007
110 Kaufmann Center
Outline

1. Basic Financial Assets and Related Issues

2. BT 1.4: Derivatives
   - The Basics
   - Black-Scholes
   - European Options
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The Basics
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1 Basic Financial Assets and Related Issues

2 BT 1.4: Derivatives
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   - European Options
Onward to Black-Scholes

At this point, let’s take some time and cruise through the stochastic processes section of our ProbStatLectures (at least up to the Stochastic Integrals section.) Highlights:

**Given Wiener process** $X(t)$, smooth function $f(X, t)$:

- **(Ito) $df = \sigma S \frac{\partial f}{\partial S} dW + \left( \mu S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \frac{\partial f}{\partial t} \right) dt$.**
- Without randomness, the term $\frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2}$ would be absent.
- Integrals with respect to $dW$ are given as stochastic process $Y(t)$, with $Y(t) - Y(0) = \int_0^t h(W(\tau), \tau) dW(\tau)$ and

$$Y(t) - Y(0) = \lim_{m \to \infty} \sum_{j=0}^{m} h(W(t_j), t_j) (W(t_{j+1}) - W(t_j)).$$
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We can actually do some simple calculations with these ideas:

Calculate the following at the board:

\[ \int_0^t dW(\tau) \]

\[ \int_0^t W(\tau) dW(\tau) \quad \text{... well, we won’t work it out, but let’s see why the obvious answer (what is it?) is wrong by calculating expectations.} \]

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Risky asset price $S(t)$:

- Is a random variable for each time $t$.
- Is described as a random process $\frac{dS}{S} = \sigma \, dW + \mu \, dt$ where $\sigma \, dW$ is the risky part and $\mu \, dt$ is the risk-free part. Discuss volatility $\sigma$ and drift $\mu$.
- If $f(S, t)$ is the price of a call or put, Ito’s Lemma tells us that

$$df = \sigma S \frac{\partial f}{\partial S} \, dW + \left( \mu S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \frac{\partial f}{\partial t} \right) \, dt.$$
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Here’s the scenario: Consider a portfolio consisting of one option at a price of $f(S,t)$ and $\Delta$ short shares of the corresponding stock at price $S$. So the value of the portfolio is

$$V = f(S,t) - \Delta \cdot S$$

**Black-Scholes Derivation:**

Analyze the differential of the price:

- $dV = df - \Delta \cdot dS$. So use Ito:

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Recall that \( dS = S\sigma dW + \mu Sdt \), so plug in, choose \( \Delta = \frac{\partial f}{\partial S} \), equate to the risk-free differential, and obtain the celebrated Black-Scholes equation, which first appeared in a 1973 paper in the Journal of Political Economy by Fischer Black and Myron Scholes titled “The pricing of options and corporate liabilities”.

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\frac{\partial f}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + rS \frac{\partial f}{\partial S} - rf = 0.
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Discussion:

Notice that randomness \( dX \) is gone! So is drift \( \mu \)! There are other aspects to this problem. Consider, e.g., a call.

- Final conditions: \( f(S, T) = \max(S - X, 0) \)
- Boundary conditions: \( f(0, T) = 0, \lim_{S \to \infty} f(S, T) = S - X \)
- There is a unique solution to this problem and we have formulas for it in the case of European options!
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Solution to Black-Scholes

For a European call:

- \( C(S, t) = SN(d_1) - Ke^{-r(T-t)}N(d_2) \)

  where

\[
N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-s^2/2} \, ds
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d_1 = \ln \left( \frac{S}{K} \right) + \left( r + \sigma^2/2 \right) (T - t) / \sigma \sqrt{T - t}
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d_2 = d_1 - \sigma \sqrt{T - t}
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- Using call-put parity, on obtains

\[P(S, t) = Ke^{-r(T-t)}N(-d_2) - SN(-d_1).\]
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The Greeks

Tools for Financial Analysis:

- \( \Delta = \frac{\partial f(S, t)}{\partial S} \): “delta” measures sensitivity of portfolio to small variations in the stock price (analogous to duration in bonds.)

- \( \Theta = \frac{\partial f(S, t)}{\partial t} \): “theta” measures sensitivity of portfolio to small variations in time (useful as expiry nears.)

- \( \Gamma = \frac{\partial^2 f(S, t)}{\partial S^2} \): “gamma” measures sensitivity of portfolio to smaller effects (analogous to convexity in bonds.)

- \( \nu = \frac{\partial f(S, t)}{\partial \sigma} \): the “vega” measures sensitivity to volatility.

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Simulating a Random Walk:

This is easy with Matlab. For example, the random walk

\[ dS = \sigma S dX + \mu S dt \]

\( \mu = 0.07 \) and \( \sigma = 0.03 \), \( S(0) = 100 \). Do the following Matlab commands.

```matlab
>mu = 0.06
>sigma = 0.03
>s = zeros(53,1);
>s(1)=100;
>dt = 1/52;
>dx = sqrt(dt)*randn(52,1);
>for ii=1:52,s(ii+1) = s(ii)+s(ii)*(sigma*dx(ii)+mu*dt);end
>plot(s), hold on, grid % now repeat experiment
```