

JDEP 384H: Numerical Methods in Business

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Lecture 11, February 13, 2007
110 Kaufmann Center

Outline

- 1 Basic Financial Assets and Related Issues
- 2 BT 1.4: Derivatives
 - The Basics
 - Black-Scholes
 - European Options

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Onward to Black-Scholes

At this point, let's take some time and cruise through the stochastic processes section of our ProbStatLectures (at least up to the Stochastic Integrals section.) Highlights:

Given Wiener process $X(t)$, smooth function $f(X, t)$:

- (Ito) $df = \sigma S \frac{\partial f}{\partial S} dW + \left(\mu S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \frac{\partial f}{\partial t} \right) dt$.
- Without randomness, the term $\frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2}$ would be absent.
- Integrals with respect to dW are given as stochastic process $Y(t)$, with $Y(t) - Y(0) = \int_0^t h(W(\tau), \tau) dW(\tau)$ and

$$Y(t) - Y(0) = \lim_{m \rightarrow \infty} \sum_{j=0}^m h(W(t_j), t_j) (W(t_{j+1}) - W(t_j)).$$

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We can actually do some simple calculations with these ideas:

Calculate the following at the board:

- $\int_0^t dW(\tau)$
- $\int_0^t W(\tau) dW(\tau)$... well, we won't work it out, but let's see why the obvious answer (what is it?) is wrong by calculating expectations.
- What is the least adjustment that gives a plausible answer?

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Risky asset price $S(t)$:

- Is a random variable for each time t .
- Is described as a random process $\frac{dS}{S} = \sigma dW + \mu dt$ where σdW is the risky part and μdt is the risk-free part. Discuss volatility σ and drift μ .
- If $f(S, t)$ is the price of a call or put, Ito's Lemma tells us that

$$df = \sigma S \frac{\partial f}{\partial S} dW + \left(\mu S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \frac{\partial f}{\partial t} \right) dt.$$

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The Equation

Here's the scenario: Consider a portfolio consisting of one option at a price of $f(S, t)$ and Δ short shares of the corresponding stock at price S . So the value of the portfolio is

$$V = f(S, t) - \Delta \cdot S$$

Black-Scholes Derivation:

Analyze the differential of the price:

- $dV = df - \Delta \cdot dS$. So use Ito:



$$dV = \sigma S \frac{\partial f}{\partial S} dW + \left(\mu S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \frac{\partial f}{\partial t} \right) dt - \Delta \cdot dS.$$

- If the portfolio were risk-free, we would have $dV = Vr dt$
- And then a miracle happens!

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Recall that $dS = S\sigma dW + \mu Sdt$, so plug in, choose $\Delta = \partial f / \partial S$, equate to the risk-free differential, and obtain the celebrated Black-Scholes equation, which first appeared in a 1973 paper in the Journal of Political Economy by Fischer Black and Myron Scholes titled “The pricing of options and corporate liabilities”.

$$\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + rS \frac{\partial f}{\partial S} - rf = 0.$$

Discussion:

Notice that randomness dX is gone! So is drift μ ! There are other aspects to this problem. Consider, e.g., a call.

- Final conditions: $f(S, T) = \max(S - X, 0)$
- Boundary conditions: $f(0, T) = 0$, $\lim_{S \rightarrow \infty} f(S, T) = S - X$
- There is a unique solution to this problem and we have formulas for it in the case of European options!
- There are important quantities associated with the formula, called the “Greeks”.

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Solution to Black-Scholes

For a European call:

- $C(S, t) = SN(d_1) - Ke^{-r(T-t)}N(d_2)$
- where

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-s^2/2} ds$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

- Using call-put parity, one obtains

$$P(S, t) = Ke^{-r(T-t)}N(-d_2) - SN(-d_1).$$

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The Greeks

Tools for Financial Analysis:

- $\Delta = \frac{\partial f(S, t)}{\partial S}$: “delta” measures sensitivity of portfolio to small variations in the stock price (analogous to duration in bonds.)
- $\Theta = \frac{\partial f(S, t)}{\partial t}$: “theta” measures sensitivity of portfolio to small variations in time (useful as expiry nears.)
- $\Gamma = \frac{\partial^2 f(S, t)}{\partial S^2}$: “gamma” measures sensitivity of portfolio to smaller effects (analogous to convexity in bonds.)
- $\nu = \frac{\partial f(S, t)}{\partial \sigma}$: the “vega” measures sensitivity to volatility.
- $\rho = \frac{\partial f(S, t)}{\partial r}$: the “rho” measures sensitivity to change in interest rate.

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Simulating a Random Walk:

This is easy with Matlab. For example, the random walk $dS = \sigma SdX + \mu Sdt$ $\mu = 0.07$ and $\sigma = 0.03$, $S(0) = 100$. Do the following Matlab commands.

```
>mu = 0.06
>sigma = 0.03
>s = zeros(53,1);
>s(1)=100;
>dt = 1/52;
>dx = sqrt(dt)*randn(52,1);
>for ii=1:52,s(ii+1) =
s(ii)+s(ii)*(sigma*dx(ii)+mu*dt);end
>plot(s), hold on, grid % now repeat experiment
```