JDEP 384H: Numerical Methods in Business

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Lecture 9, February 6, 2007
110 Kaufmann Center

Outline

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Linear Programming Example

Example

Solve the following linear programming problem graphically:
Minimize the (linear) objective function

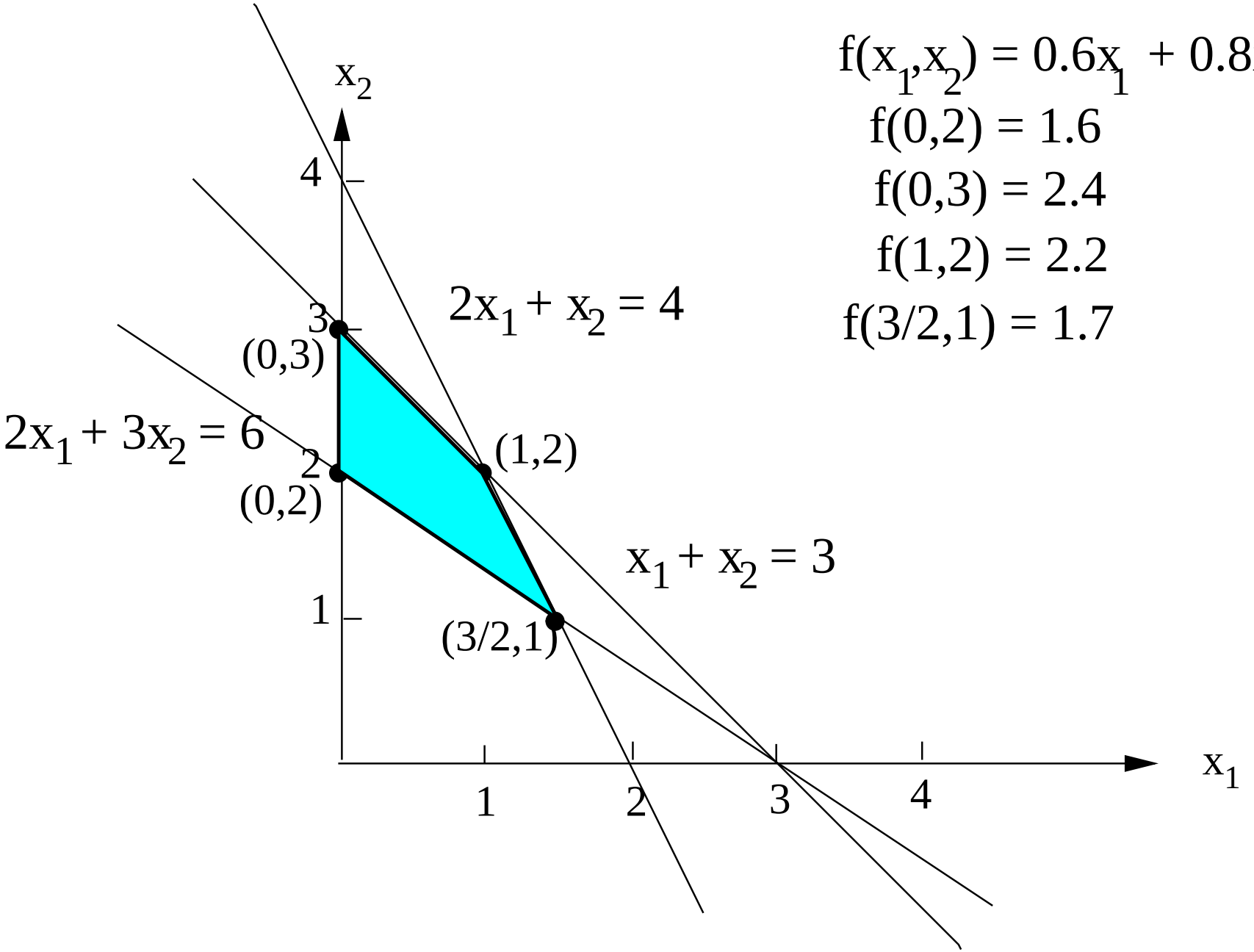
$$f(x_1, x_2) = 0.6x_1 + 0.8x_2$$

subject to the constraints

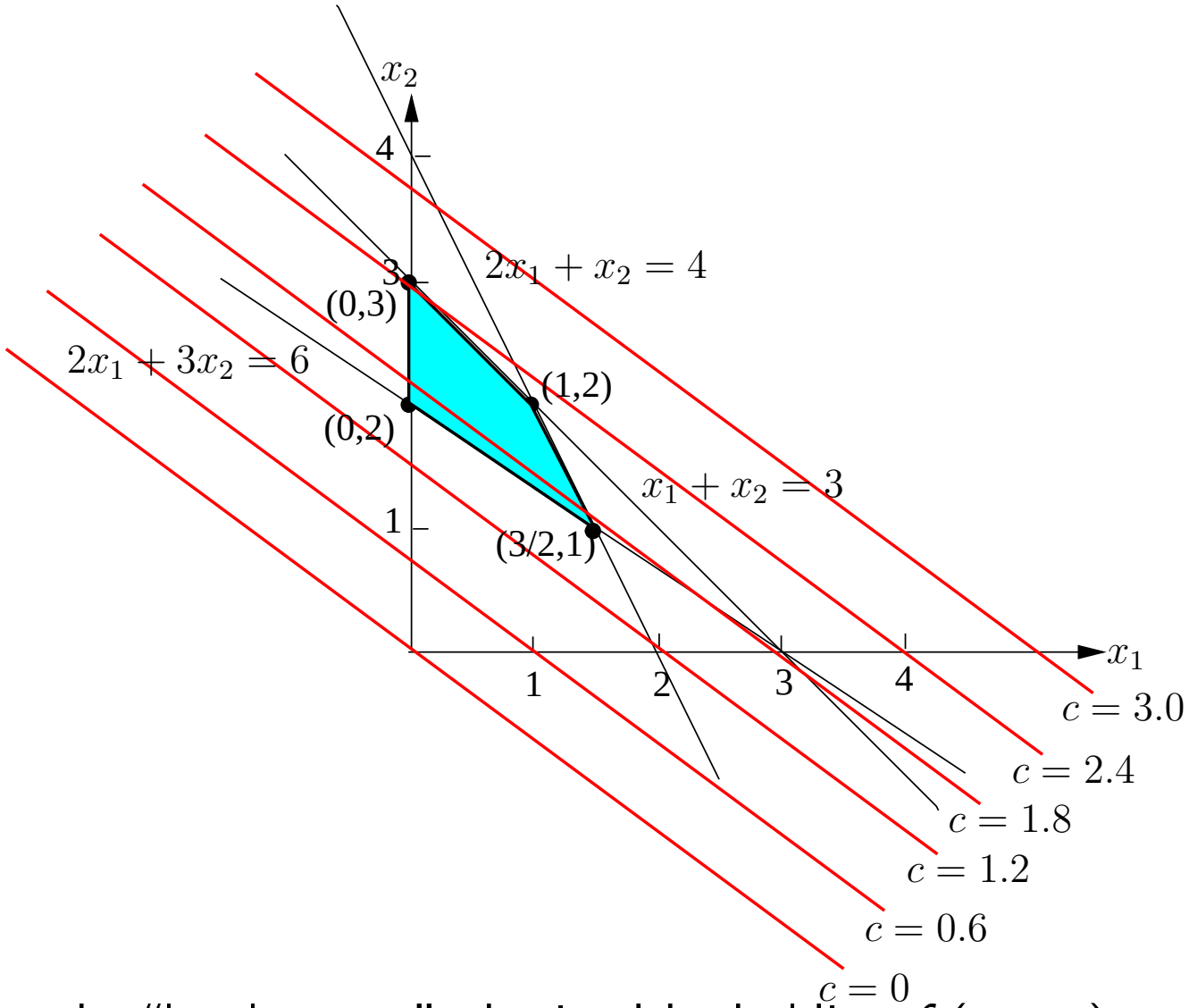
$$\begin{aligned} x_1 + x_2 &\leq 3 \\ 2x_1 + x_2 &\leq 4 \\ 2x_1 + 3x_2 &\geq 6 \\ x_i &\geq 0,\ i = 1, 2. \end{aligned}$$

Solution. Examine the graph on the next slide and discuss it. Note the original variables, x_1, x_2 in this case, are called **decision variables**.

We Were Here


$f(x_1,x_2) = 0.6x_1 + 0.8x_2$
$f(0,2) = 1.6$
$f(0,3) = 2.4$
$f(1,2) = 2.2$
$f(3/2,1) = 1.7$
x_2
x_1
$2x_1 + x_2 = 4$
$2x_1 + 3x_2 = 6$
$x_1 + x_2 = 3$
(0,3)
(1,2)
(0,2)
(3/2,1)
1
2
3
4

Why Does This Work?



Examine the "level curves" obtained by holding $f(x_1, x_2)$ constant. These are lines $c = 0.6x_1 + 0.8x_2$ of slope $0.6/0.8 = 3/4$ that move to the right as c increases with x_1-intercept $c/0.6$.

Linear Programming Example

Work the expression parts of these problems out at the board (we may need to review terms in LinearAlgebraLecture-384h notes):

With reference to preceding problem:

- Express the problem in canonical form using matrices.
- Use the Matlab linprog to solve this problem.
- Express the problem in standard form using matrices.
- Solve the problem with the Matlab program lp.

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A Classic Example

For applications, it isn't the MATH that is the goal, it's the MODEL, that is, word problems are more important than formulas! A classic example the standard text in OR (Hillier and Lieberman, Introduction to Operations Research, eighth edition):

Wyndor Glass Company:

- Produces high-quality glass products, including windows and glass doors.
- Has 3 plants: plant 1 produces aluminum frames and hardware, plant 2 produces wood frames and plant 3 produces the glass and finishes the products.
- Two new products are proposed, an 8-foot glass door (product 1) and a 4x6 double hung wood-framed door (product 2).
- These will be produced in batches of 20, and the problem is to determine the production rates that will maximize profit, given the data that has been collected in the following tables.

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Wyndor Data

Here is the data, carefully collected by the OR team:

	Production per Batch	Time (Hours)	Available Production
Plant	Product 1	Product 2	Time per Week (Hours)
1	1	0	4
2	0	2	12
3	3	2	18
Profit per batch	\$3K	\$5K	

Now formulate this problem in canonical and standard form.

Outline

Immunization Strategies

Example

Use Matlab to determine the correct weighting of three bonds with durations $2, 4, 6$ and convexities $12, 15, 20$, respectively, if we are to shape a portfolio with duration 3 and convexity 13.

Solution. Use the First Pass strategy. Work this system out with Matlab.

What about the Second Pass?

With 3 bonds, we're stuck. But increase the number by, say one, to 4 bonds. Now we have a new situation of 3 equations in 4 unknowns. Since unknowns exceed equations, we can expect infinitely many solutions if any at all (see LinearAlgebraLecture)!

So which do we select?