JDEP 384H: Numerical Methods in Business

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110 Kaufmann Center

Outline

Portfolio Allocation:

Say n assets are available, with returns R_i, $i = 1, \ldots, n$, each with expected value $E[R_i]$. A weighted portfolio with no short positions, $R = w_1 R_1 + \cdots + w_n R_n$ should give return

$$E[R] = w_1 E[R_1] + \cdots + w_n E[R_n].$$

Maximizing return is easy! How? Is this a good idea?

A Binomial Model:

a portfolio is to be constructed from two assets. The first is risk-free at a price of \$1 and offering a 10% growth rate in one time period. The other is risky at the same price and its value in the next time period could be \$1.20 or \$1.40. We don't expect this situation to last long? Why not?

- Rule for binomial models: if the current risk-free rate is r_f and a binomial model for a risky asset has multiplicative shocks d and u, then

$$d < 1 + r_f < u$$

Outline

1. Basic Financial Assets and Related Issues

2. BT 2.3: Fixed Income Securities
   - Basic Theory of Interest Rates
   - Interest Rate Sensitivity

Some Notes on Bonds

- In the financial world, the default period for bonds is 2, which means two payments per year; periods are specified on a per year basis.
- In the financial world individual bond face values are typically quoted at $1000 rather than $100.
- Matlab's financial functions are *SIA compliant*, which means that they adhere to the definitions and standards set by the Securities Industry Association.

Examples

Example

(simplest) zero coupon bond with period and maturity of one year. This security has a purchase price P and face value (a.k.a. par value) F, which is paid at maturity. It has a *return* $R = F/P$ and rate of return $r = R - 1$ ($100r$ as a percent) so that

$$F = (1 + r) P.$$

Example

(not so simple): coupon bond with price P, face (or par) value F, coupon rate r (remember to scale the rate appropriately if it is given as an annual percentage and the period is not a year) and maturity of n periods. At the end of each period, a payment (coupon) of rF is made and additionally, at maturity (end of last period) the face value F is added to the payment.

Valuing Fixed-Income Securities

Suppose it generates a cash flow stream

$$[c_1, c_2, \ldots, c_n]$$

of payouts at the end of the first through nth periods.

***Present value* of this stream with a discount rate r:**

$$PV = \sum_{k=1}^{n} \frac{c_k}{(1+r)^k}.$$

Meaning: an initial investment of PV that could earn money at the rate r would exactly cover the cash flow stream of the security. Since someone pays a price P for the security, a better cash flow stream:

$$[c_0, c_1, c_2, \ldots, c_n]$$

where $c_0 = -P$.

Yield of Security

From this point of view, we ask: What discount rate would make the net present value of the contract+cost equal to zero? Just extend the formula above to include a zero-th payment and set the result equal to zero.

$$0 = \sum_{k=0}^{n} \frac{c_k}{(1+r)^k} \quad \text{or } P = \sum_{k=1}^{n} \frac{c_k}{(1+r)^k}.$$

The rate that solves this equation is called the ***yield*** of the security. How do we find the yield? Easy: just set

$$h = \frac{1}{1+r}$$

and rewrite the equation as

$$0 = \sum_{k=0}^{n} c_k h^k.$$

Now solve this polynomial equation, find (hopefully) only one real root h, and solve to get $r = \frac{1}{h} - 1$.

Example

A coupon bond has maturity of 5 years, period of a year, face value of 100 and coupon rate of 7%.
(a) Use Matlab to find the present value of the bond at a discount rate of 6%.
(b) At a price $P = 110$, find the yield of the bond.
(c) At a price of $P = 100$, find the yield of the bond.

Solution. Here's the Matlab code that does the job:

```
help pvvar
help irr % help irroct for octave users
r = 0.06
cf = [0, 7, 7, 7, 7, 107]
pvvar(cf,r)
cf(1)=-110
irr(cf)
cf(1)=-100
irr(cf)
```

The Term Structure of Interest Rates

Our models are simplified versions of reality (no surprise!). As a matter of fact:

- Interest rates vary with time.
- Market folks have a name for this: the **term structure of interest rates**, by which they mean the way in which the yield of a security (interest and redemption value) varies according to the term of the security.
- The normal picture is that longer terms lead to higher rates.
- However, to some extent, the graph of the term structure (let's put some on the board out 20 years) reflects market expectations of the future. So the formulas for present value should be modified ($r = r(t)$) for a more realistic model.
- Upward rates occur in a growing economy in anticipation of higher interest rates.
- Downward rates occur in an economy approaching recession, where lower interest rates are anticipated.

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In-between Bonds

Pricing a bond purchased in-between coupon payment dates: You (the buyer) get the benefit of the next coupon payment, even though you have not waited for a full period. Therefore, you

- calculate "clean price", that is, the price you would have paid had you purchased it one full period before the next coupon payment,
- then add to the clean price the payment of the fraction of the next coupon that the seller has earned by holding the bond to the settement date (date on which you purchased the bond.) The resulting "dirty price" is what you will pay for the bond.

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In-between Bond Example

Example

A coupon bond is issued on December 31, 2001, with a maturity date of December 31, 2003, face value \$100, coupon rate of 8%, yield of 10%, with period of 2 coupons per year. You purchase this bond on August 15, 2002. What are the clean and dirty prices of the bond on that date?

Solution. We compute compute the clean price by assuming that we had purchased the bond on the first coupon date prior to our actual purchase date (of course, we don't get that coupon.) Thus, we will receive the three remaining coupons. We then compute the fraction of days between that coupon date and the settlement (purchase) date divided by the number of days between that coupon date and the first coupon date following settlement.

All of this can be done easily in Matlab:

```
cleanprice = pvvar([0 4 4 104])
prorate =
(datenum('10-aug-2002')-datenum('30-jun-2002'))/ ...
(datenum('31-dec-2002') - datenum('30-jun-2002'))
dirtyprice = cleanprice + prorate*4
```

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Price-yield Curve

Fact of Life:

Interest rates change.

Suppose:

- You purchase a four year coupon bond with a yearly period and rate of return equal to the current risk-free interest rate with matching face value and price of $1000.
- Immediately after collecting the second coupon, you want to sell the bond for some cash.
- But rates have changed! What will you get for the bond?
- How would an increase of one percentage affect the new price?
- A useful tool is the **price-yield curve**, $P(\lambda)$, which is the present value of the security at discount rate λ, and typically resembles what? Let's discuss what the curve should look like and sketch one.

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Taylor Approximations

Recall from calculus: given smooth function $f(\lambda)$

Linear approximation: $\delta f \equiv f(\lambda + \delta\lambda) - f(\lambda) \approx \frac{df}{d\lambda}(\lambda)\,\delta\lambda$

Quadratic approximation: $\delta f \approx \frac{df}{d\lambda}(\lambda)\,\delta\lambda + \frac{d^2 f}{d\lambda^2}(\lambda)\,\frac{(\delta\lambda)^2}{2}$

Questions:

- What does $df(\lambda)/d\lambda$ measure?
- Answer: the rate of change of the function f at the argument λ.
- What is the significance of a large $d^2 f(\lambda)/d\lambda^2$?
- Answer: Answer: increased curvature at λ.

Use Matlab to illustrate these approximations for $f(\lambda) = e^{\lambda}$ near $\lambda = 0$.

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Cash Flow Calculations

Given a cash flow

$$[0, c_1, c_2, \ldots, c_n]$$

of period m priced at a yield λ, we have these standard terms:

- **(Modified) Duration:** $D_M = -\frac{1}{P}\frac{dP}{d\lambda}$
- **Duration:** $D = (1 + \lambda/m)\, D_M$
- **Convexity:** $C = \frac{1}{P}\frac{d^2 P}{d\lambda^2}$
- All of which yields: $\delta P \approx -D_M\, P\, \delta\lambda + \frac{P\, C}{2}(\delta\lambda)^2$
- This suggests: if you *own* this cash flow, a larger convexity is a good thing. Why?

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Since k/m is the number of periods before c_k is paid out, i.e., the cash flow time for c_k, we can interpret D as a weighted average over present values of cash flow times. Note duration of a zero coupon bond is just time to maturity.

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Tools for Calculations

We have Matlab tools for computing duration and convexity.

Example

A coupon bond has face value of \$100, rate of 6%, period of six months and maturity of three years. Price the bond at a yield (=discount rate) of 7% and find the change in price if yield is increased to 7.5%. Compare to the first and second-order (quadratic) approximation at $\lambda = 0.07$.

Solution. Let's do it with Matlab, using the functions pvvar, cfdurm and cfconv. Here's a start:

```
>cf = [0, 3, 3, 3, 3, 3, 103]
>p1 = pvvar(cf,0.07)
>p2 = pvvar(cf,0.08)
>deltaf = p2 - p1
>deltalam = 0.005
>% keep on working...
```