1. The Calculus
   - Rates of Change and Derivatives
   - Area and Integrals
   - Multivariate Calculus
Let’s start this lecture by opening the file CalculusLecture-384H.pdf and following along the sections as we cover them here. Here is the example we’ll be dissecting:

**Example**

Let \( D = [0, 5] \), the interval of real numbers \( x \in \mathbb{R} \) such that \( -\pi \leq x \leq \pi \), and define a function \( f : D \to \mathbb{R} \) by the formula

\[
f(x) = \frac{3}{1 + x^2} + 2x.
\]
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Derivative as Rate of Change

Example

Find the derivative $f'(x)$ of $f(x)$ and plot them together with Matlab. Then plot the differences $\Delta f / \Delta x$ with $x = 0.1$ and see how well they do on this interval.

The Matlab code:

```matlab
x = 0:.01:5;
f = @(x) 3./(1 + x.^2) + 2*x;
fp = @(x) -6*x./(1 + x.^2).^2 +2;
plot(x,f(x))
grid, hold on
plot(x,fp(x))
dx = 0.5
plot(x,(f(x+dx)-f(x))/dx)
```

Try this last line again with $dx = 0.1$. 
Example

Use the calculations of the previous example to approximate $f(2)$ and $f(4)$ using differentials and the values of $f, f'$ at $x = 3$. Also try the quadratic approximation supplied by Taylor’s polynomials.

Here is the Matlab code:

```
a = 3
x = 4
f(x)
f(a) + fp(a)*(x-a)
fpp = @(x) 6*(3*x.^2-1)./(1+x.^2).^3
f(a) + fp(a)*(x-a) + 1/2*fpp(a)*(x-a)^2
x = 2
f(x)
f(a) + fp(a)*(x-a)
f(a) + fp(a)*(x-a) + 1/2*fpp(a)*(x-a)^2
```
Outline

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Example

Let $f(x)$ be as in the first example, and calculate \( \int_0^4 f(x) \, dx \) with Matlab by using the FTOC and a Riemann sum as in the notes. Also verify the comment about left and right Riemann sums.

Here is the Matlab code that does it.

```matlab
F = @(x) 3*atan(x) + x.^2
F(4)-F(0)
dx = 0.1
x = 0:dx:4;
% Riemann sum approximation to area
Rleft = dx*sum(f(x(1:length(x)-1)))
Rright = dx*sum(f(x(2:length(x))))
Raverage = 0.5*(Rleft + Rright)
```
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Example

Given that $f(x, y) = (x^2 + y^3)^{3/2}$, find a formula for the differential $df$ and in particular, for the differential evaluated at $x = 2, y = 1$. How does this help you describe the tangent plane approximation to $z = f(x, y)$ for $(x, y)$ near the point $(1, 2)$? Test this formula at $(0.8, 2.1)$.

Work this at the board and check arithmetic with Matlab:

$x = 0.8, \ y = 2.1$
$a = 1, \ b = 2$
$dx = x-a, \ dy = y-b$
$fab = (a^2 + b^3)^{1.5}$
$fx = 1.5*(a^2+b^3)^{0.5}*2*a$
$fy = 1.5*(a^2+b^3)^{0.5}*3*b^2$
$dz = fx*dx + fy*dy$
$fapprox = fab + dz$
$factual = (x^2 + y^3)^{1.5}$