JDEP 384H: Numerical Methods in Business

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Outline

Let's start this lecture by opening the file CalculusLecture-384H.pdf and following along the sections as we cover them here. Here is the example we'll be dissecting:

Example

Let $D = [0, 5]$, the interval of real numbers $x \in \mathbb{R}$ such that $-\pi \leq x \leq \pi$, and define a function $f : D \to \mathbb{R}$ by the formula

$$f(x) = \frac{3}{1 + x^2} + 2x.$$

Outline

Example

Find the derivative $f'(x)$ of $f(x)$ and plot them together with Matlab. Then plot the differences $\Delta f/\Delta x$ with $x = 0.1$ and see how well they do on this interval.

The Matlab code:

```
x = 0:.01:5;
f = @(x) 3./(1 + x.^2) + 2*x;
fp = @(x) -6*x./(1 + x.^2).^2 +2;
plot(x,f(x))
grid, hold on
plot(x,fp(x))
dx = 0.5
plot(x,(f(x+dx)-f(x))/dx)
```

Try this last line again with $dx = 0.1$.

Differential Approximation

Example

Use the calculations of the previous example to approximate $f(2)$ and $f(4)$ using differentials and the values of f, f' at $x = 3$. Also try the quadratic approximation supplied by Taylor's polynomials.

Here is the Matlab code:

```
a = 3
x = 4
f(x)
f(a)+fp(a)*(x-a)
fpp = @(x) 6*(3*x.^2-1)./(1+x.^2).^3
f(a)+fp(a)*(x-a) + 1/2*fpp(a)*(x-a)^2
x = 2
f(x)
f(a)+fp(a)*(x-a)
f(a)+fp(a)*(x-a) + 1/2*fpp(a)*(x-a)^2
```

Outline

1. The Calculus
   - Rates of Change and Derivatives
   - Area and Integrals
   - Multivariate Calculus

Definite Integrals as Area

Example

Let $f(x)$ be as in the first example, and calculate $\int_0^4 f(x)\, dx$ with Matlab by using the FTOC and a Riemann sum as in the notes. Also verify the comment about left and right Riemann sums.

Here is the Matlab code that does it.

```
F = @(x) 3*atan(x) + x.^2
F(4)-F(0)
dx = 0.1
x = 0:dx:4;
% Riemann sum approximation to area
Rleft = dx*sum(f(x(1:length(x)-1)))
Rright = dx*sum(f(x(2:length(x))))
Raverage = 0.5*(Rleft + Rright)
```

Outline

Example

Given that $f(x,y) = \left(x^2 + y^3\right)^{3/2}$, find a formula for the differential df and in particular, for the differential evaluated at $x = 2$, $y = 1$. How does this help you describe the tangent plane approximation to $z = f(x,y)$ for (x,y) near the point $(1,2)$? Test this formula at $(0.8, 2.1)$.

Work this at the board and check arithmetic with Matlab:

```
x = 0.8, y = 2.1
a = 1, b = 2
dx = x-a, dy = y-b
fab = (a^2 + b^3)^1.5
fx = 1.5*(a^2+b^3)^0.5*2*a
fy = 1.5*(a^2+b^3)^0.5*3*b^2
dz = fx*dx + fy*dy
fapprox = fab + dz
factual = (x^2 + y^3)^1.5
```