JDEP 384H: Numerical Methods in Business

Instructor: Thomas Shores
Department of Mathematics

Lecture 26, April 19, 2007
110 Kaufmann Center

Outline

Schedule for Dead Week

Our Schedule:

- Tuesday, April 24: Finish course with examples from game theory and decision analysis.
- Wednesday, April 25: Official due date for Assignment 5, though I will accept homework on Thursday, April 26, without penalty.
- Thursday, April 27: Discuss the final exam and do in-class course evaluations. In addition, you should do on-line evaluations, about which you should have been notified by email.
- Tuesday, May 1: Final Exam in 110 Kaufmann Center.

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1. NT: Decision Analysis and Game Theory
   - An Intelligent Opponent: Game Theory
   - An Indifferent Opponent: Nature
   - Decision Making Without Experimentation
   - Decision Making with Experimentation

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A Model Problem

The Problem:

A small game company, Sixth Degree, has invested considerable effort in preliminary development on a game concept that they believe holds promise. The showed a preliminary prototype at the annual game developer trade show E3 in March.

- Subsequently they found producers who wants to purchase the IP for \$850,000 (the best offer) and continue development without further involvement with Sixth Degree.
- They were also encouraged by some producers to develop a full working prototype and then sell the IP to the producers with a better purchase price and some handsome royalty arrangements.
- A decision has to be made, i.e., a pure strategy has to be selected, and a mixed strategy won't do as a substitute. What

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Model Problem Data

The Data:

- SD estimates the cost of further development to be about one million dollars.
- If the working prototype were accepted by one of the major producers, SD estimates that total profit from sale of the IP and negotiated royalties to be about seven million dollars.
- SD estimates the probability of this game being accepted at about 1/4.
- The data in "payoff table" form:
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	States of Nature	
Alternatives	Acceptable	Unacceptable
Develop IP	$\$7M$	$-\$1M$
Sell IP	$\$0.85M$	$\$0.85M$
Prior Probabilities	0.25	0.75

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Maximin Strategy

Solution:

The idea is to look at the worst outcomes for each alternative, then choose the most favorable of worst payoffs. Since nature is not really a player, this only pertains to the company SD.

- Let's work this example out at the board.
- Problem with this strategy: It makes sense when one is competing against a rational and malevolent opponent. Nature isn't.
- Another problem: It ignores additional information (the probabilities), so is a very conservative choice.

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Maximum Likelihood Strategy

Solution:

Identify the most likely state of nature. From this state, find the decision alternative with the maximum payoff.

- Let's work this example out at the board.
- Problem with this strategy: Although still accounting for all the data, it gives excessive weight to one piece of the data – the most likely state. What if there are states that are close in likelihood?
- So again it is a conservative choice whose value might diminish considerably if the prior probabilities are very far off.

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Bayes' Decision Rule

Solution:

Calculate the expected value of the payoff for each alternative using the best available estimates of the probabilities of the states of nature.

- Let's work this example out at the board.
- Advantage: This strategy accounts for all the data and gives some weight to states that are not the most likely.
- Advantage: This strategy is amenable to a sensitivity analysis in terms of the prior probabilities. Let's make a sensitivity graph of the decision regions based on the prior probability p of acceptable state. Plot expectation with each decision against p, $0 \leq p \leq 1$. We should make a payoff matrix, priors vector, and calculate the expected payoffs

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An Experiment

The Experiment:

SD also made contact with a consulting firm, Game Development Consultants, that specializes in game business issues and has many high level contacts in the business.

- They could be hired to conduct a feasibility study of SD's plans and estimate the probability of success, i.e., acceptable state in the case of development.
- Their success rates are no secret. In fact, GDC uses them to advertise their services. In situation such as SD finds itself, they made an favorable recommendation in 60% of the cases where product was developed and successful, and an unfavorable recommendation 80% of the cases where the not developed.
- The fee for this study is $30,000. What should SD do?

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Probability Background

The relevant tabular data:

Consultant Recommendations	States of Nature: Acceptable	Unacceptable
Develop	0.6	
Sell		0.8

- This table is equivalent to a table of conditional probabilities as follows. Call the matrix below C for conditional probability:

Consultant Recommendations	States of Nature: Acceptable	Unacceptable
Develop	$P(D \mid A)$	$P(D \mid U)$
Sell	$P(S \mid A)$	$P(S \mid U)$

- We're interested in posterior probabilities $P(A \mid D)$, etc. These probabilities are backwards, aren't they? What to do?

- In fact, should we experiment at all? What would be the expected value of perfect information (EVPI) = expected payoff with perfect information - expected payoff without experimentation?

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Conditional Probabilities

Facts that we will need in this decision analysis:

- **Law of Total Probability:** Given disjoint and exhaustive events $E_1, E_2, \ldots, E_n$, and another event F,

$$P(F) = \sum_{i=1}^{n} P(F \mid E_i)\, P(E_j)$$

- **Bayes' Theorem (Short Form):**

$$P(E \mid F) \equiv \frac{P(F \mid E)\, P(E)}{P(F)}.$$

- **Bayes' Theorem (Long Form):** With same notation and hypotheses as Law of Total Probability:

$$P(E_k \mid F) \equiv \frac{P(F \mid E_k)\, P(E_k)}{\sum_{i=1}^{n} P(F \mid E_i)\, P(E_j)}.$$

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Probability Connections

The table (or matrix Q for posterior probabilities) that we want:

State of Nature	Consultant Recommends	
	Develop	Sell
Acceptable	$P(A \mid D)$	$P(A \mid S)$
Unacceptable	$P(U \mid D)$	$P(U \mid S)$

- In matrix form Q can be calculated from Bayes' theorem as

$$\begin{bmatrix} P(A \mid D) & P(A \mid S) \\ P(U \mid D) & P(U \mid S) \end{bmatrix} = \begin{bmatrix} \frac{P(D \mid A)P(A)}{P(D)} & \frac{P(S \mid A)P(A)}{P(S)} \\ \frac{P(D \mid U)P(U)}{P(D)} & \frac{P(S \mid U)P(U)}{P(S)} \end{bmatrix}$$

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$$\begin{bmatrix} P(A) & 0 \\ 0 & P(U) \end{bmatrix} \begin{bmatrix} P(D \mid A) & P(D \mid U) \\ P(S \mid A) & P(S \mid U) \end{bmatrix}^T \begin{bmatrix} \frac{1}{P(D)} & 0 \\ 0 & \frac{1}{P(S)} \end{bmatrix}$$

- and by the law of total probability

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$$\begin{bmatrix} P(A \mid D) & P(A \mid S) \\ P(U \mid D) & P(U \mid S) \end{bmatrix} = \begin{bmatrix} \frac{P(D \mid A)P(A)}{P(D)} & \frac{P(S \mid A)P(A)}{P(S)} \\ \frac{P(D \mid U)P(U)}{P(D)} & \frac{P(S \mid U)P(U)}{P(S)} \end{bmatrix}$$

- $=$

$$\begin{bmatrix} P(A) & 0 \\ 0 & P(U) \end{bmatrix} \begin{bmatrix} P(D \mid A) & P(D \mid U) \\ P(S \mid A) & P(S \mid U) \end{bmatrix}^T \begin{bmatrix} \frac{1}{P(D)} & 0 \\ 0 & \frac{1}{P(S)} \end{bmatrix}$$

- and by the law of total probability

$$\begin{bmatrix} P(D) \\ P(S) \end{bmatrix} = \begin{bmatrix} P(D \mid A) & P(D \mid U) \\ P(S \mid A) & P(S \mid U) \end{bmatrix} \begin{bmatrix} P(A) \\ P(U) \end{bmatrix}.$$

Probability Connections

The table (or matrix Q for posterior probabilities) that we want:

State of Nature	Consultant Recommends	
	Develop	Sell
Acceptable	$P(A \mid D)$	$P(A \mid S)$
Unacceptable	$P(U \mid D)$	$P(U \mid S)$

- In matrix form Q can be calculated from Bayes' theorem as

$$\begin{bmatrix} P(A \mid D) & P(A \mid S) \\ P(U \mid D) & P(U \mid S) \end{bmatrix} = \begin{bmatrix} \frac{P(D \mid A)P(A)}{P(D)} & \frac{P(S \mid A)P(A)}{P(S)} \\ \frac{P(D \mid U)P(U)}{P(D)} & \frac{P(S \mid U)P(U)}{P(S)} \end{bmatrix}$$

- $=$

$$\begin{bmatrix} P(A) & 0 \\ 0 & P(U) \end{bmatrix} \begin{bmatrix} P(D \mid A) & P(D \mid U) \\ P(S \mid A) & P(S \mid U) \end{bmatrix}^T \begin{bmatrix} \frac{1}{P(D)} & 0 \\ 0 & \frac{1}{P(S)} \end{bmatrix}$$

- and by the law of total probability

$$\begin{bmatrix} P(D) \\ P(S) \end{bmatrix} = \begin{bmatrix} P(D \mid A) & P(D \mid U) \\ P(S \mid A) & P(S \mid U) \end{bmatrix} \begin{bmatrix} P(A) \\ P(U) \end{bmatrix}.$$