JDEP 384H: Numerical Methods in Business

Instructor: Thomas Shores
Department of Mathematics

Lecture 27, April 24, 2007
110 Kaufmann Center

Outline

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Sidebar: Grades and Final

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- The grade lines are
  - A: 87%, B: 76%, C: 60%
  - I will use this for both homework and midterm.
  - Final Exam scale will be similar.

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  - Final Exam scale will be similar.
- And what about the final?
  - Will be all take-home. Check the course directory for last year's final exam key.
  - Will be available on the web in our course directory on Friday, April 27, 1:00 pm.
  - Will be due on Thursday, April 3, 1:00 pm.

Review Model Problem Data

The Payoff Table and Conditional Probability Data:

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Payoffs Alternatives	States of Nature	
	Acceptable	Unacceptable
Develop IP	$\$7M$	$-\$1M$
Sell IP	$\$0.85M$	$\$0.85M$
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- Cost of Consulting: $\$30{,}000 = \$0.03M$.

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- **Law of Total Probability:** Given disjoint and exhaustive events $E_1, E_{2,}, \ldots, E_n$, and another event F, $P(F) = \sum_{i=1}^{n} P(F \mid E_i) P(E_j)$.

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- **(Long) Bayes' Theorem:** $P(E_k \mid F) \equiv \frac{P(F|E_k)P(E_k)}{\sum_{i=1}^{n} P(F \mid E_i)P(E_j)}$.

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- and by the law of total probability unconditional probabilities are

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Decision Trees

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- Let's define a conditionals matrix and calculate the probabilities of each recommendation and then.

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Strategy		Player 2		
		1	2	3
Player 1	1	2	3	-2
	2	-1	4	0
	3	3	-2	-1

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  - Better packaging.
  - An advertising campaign.
  - Slight price reduction.
- Suppose there is considerable uncertainty about the payoff in the case that both players make a slight reduction in price. How could we clearly illustrate the effect of changes in the payoff on the weight that one of the companies puts on this strategy?

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- With this record, should we consider them?

Strategy		Actual (in percent)		
		Poor	Average	Good
Evaluation	Poor	50	40	20
	Average	40	50	40
	Good	10	10	40

Which Brings Us To...

THE END