JDEP 384H: Numerical Methods in Business

Instructor: Thomas Shores
Department of Mathematics

Lecture 26, April 19, 2007
110 Kaufmann Center
Outline

1. **NT: Decision Analysis and Game Theory**
   - An Intelligent Opponent: Game Theory
   - An Indifferent Opponent: Nature
   - Decision Making Without Experimentation
   - Decision Making with Experimentation
Our Schedule:

- Tuesday, April 24: Finish course with examples from game theory and decision analysis.
- Wednesday, April 25: Official due date for Assignment 5, though I will accept homework on Thursday, April 26, without penalty.
- Thursday, April 27: Discuss the final exam and do in-class course evaluations. In addition, you should do on-line evaluations, about which you should have been notified by email.
- Tuesday, May 1: Final Exam in 110 Kaufmann Center.
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- Subsequently they found producers who wants to purchase the IP for $850,000 (the best offer) and continue development without further involvement with Sixth Degree.
- They were also encouraged by some producers to develop a full working prototype and then sell the IP to the producers with a better purchase price and some handsome royalty arrangements.
- A decision has to be made, i.e., a pure strategy has to be selected, and a mixed strategy won’t do as a substitute. What to do?
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A Model Problem

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- SD estimates the cost of further development to be about one million dollars.
- If the working prototype were accepted by one of the major producers, SD estimates that total profit from sale of the IP and negotiated royalties to be about seven million dollars.
- SD estimates the probability of this game being accepted at about 1/4.
- The data in “payoff table” form:

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Maximin Strategy

Solution:
The idea is to look at the worst outcomes for each alternative, then choose the most favorable of worst payoffs. Since nature is not really a player, this only pertains to the company SD.

- Let’s work this example out at the board.
- Problem with this strategy: It makes sense when one is competing against a rational and malevolent opponent. Nature isn’t.
- Another problem: It ignores additional information (the probabilities), so is a very conservative choice.
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Identify the most likely state of nature. From this state, find the decision alternative with the maximum payoff.

- Let’s work this example out at the board.
- Problem with this strategy: Although still accounting for all the data, it gives excessive weight to one piece of the data – the most likely state. What if there are states that are close in likelihood?
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Calculate the expected value of the payoff for each alternative using the best available estimates of the probabilities of the states of nature.

- Let’s work this example out at the board.
- Advantage: This strategy accounts for all the data and gives some weight to states that are not the most likely.
- Advantage: This strategy is amenable to a sensitivity analysis in terms of the prior probabilities. Let’s make a sensitivity graph of the decision regions based on the prior probability $p$ of acceptable state. Plot expectation with each decision against $p$, $0 \leq p \leq 1$. We should make a payoff matrix, priors vector, and calculate the expected payoffs.
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An Experiment

The Experiment:

SD also made contact with a consulting firm, Game Development Consultants, that specializes in game business issues and has many high level contacts in the business.

- They could be hired to conduct a feasibility study of SD’s plans and estimate the probability of success, i.e., acceptable state in the case of development.
- Their success rates are no secret. In fact, GDC uses them to advertise their services. In situation such as SD finds itself, they made an favorable recommendation in 60% of the cases where product was developed and successful, and an unfavorable recommendation 80% of the cases where the not developed.
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Facts that we will need in this decision analysis:

- **Law of Total Probability**: Given disjoint and exhaustive events \( E_1, E_2, \ldots, E_n \), and another event \( F \),

  \[
P(F) = \sum_{i=1}^{n} P(F \mid E_i) P(E_i)
\]

- **Bayes’ Theorem (Short Form)**:

  \[
P(E \mid F) \equiv \frac{P(F \mid E) P(E)}{P(F)}.
\]

- **Bayes’ Theorem (Long Form)**: With same notation and hypotheses as Law of Total Probability:

  \[
P(E_k \mid F) \equiv \frac{P(F \mid E_k) P(E_k)}{\sum_{i=1}^{n} P(F \mid E_i) P(E_i)}.
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Facts that we will need in this decision analysis:

- **Law of Total Probability**: Given disjoint and exhaustive events $E_1, E_2, \ldots, E_n$, and another event $F$,
  \[
  P(F) = \sum_{i=1}^{n} P(F \mid E_i) P(E_i)
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Conditional Probabilities

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The table (or matrix $Q$ for posterior probabilities) that we want:

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<thead>
<tr>
<th>State of Nature</th>
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<tr>
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In matrix form $Q$ can be calculated from Bayes’ theorem as

$$
\begin{bmatrix}
P(A|D) & P(A|S) \\
P(U|D) & P(U|S)
\end{bmatrix} = \begin{bmatrix}
\frac{P(D|A)P(A)}{P(D)} & \frac{P(S|A)P(A)}{P(S)} \\
\frac{P(D|U)P(U)}{P(D)} & \frac{P(S|U)P(U)}{P(S)}
\end{bmatrix}
$$

$$
\begin{bmatrix}
P(D) \\
P(S)
\end{bmatrix} = \begin{bmatrix}
P(D|A) & P(D|U) \\
P(S|A) & P(S|U)
\end{bmatrix}^T \begin{bmatrix}
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