Midterm Exam JDEP 384H: Numerical Methods for Business

Name: ______________________ **Score:** __________

Instructions: Write out your solutions on the paper provided. Show your work and give reasons for your answers. NO notes, calculators, laptops, cell phones or other electronic equipment allowed. Each problem is worth 20 points *for a total of* 100 *points.* Work problem 1 and any four of the remaining five problems. *Clearly indicate which four you want graded.* Give exact answers (like $1/3$ or $\sqrt{2}$) where calculations are expected.

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1. Answer True or False for each of the following statements. If false, correct the statement.

(a) An eigenvector for the (square) matrix A is a vector $\mathbf{x}$ such that $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ called an eigenvalue of A.

(b) If D_M is the modified duration of a bond and C is its convexity at rate λ, then $\delta P = P(\lambda + \delta\lambda) - P(\lambda)$ has second order approximation $-D_M P\,\delta\lambda + \frac{PC}{2}(\delta\lambda)^2$.

(c) The correlation between two random variables X and Y with covariance C and standard deviations a, b, respectively, is $C/(ab)$

(d) The VaR of a portfolio whose returns are normally distributed is completely determined by its value, volatility, and a time horizon.

(e) For a generic option of price $f(S,t)$ the "Greek" $\Delta = \partial f(S,t)/\partial S$ measures the sensitivity of the option price to small changes in time.

(f) The price of an American call with dividends may fall below the payoff curve.

(g) In order to construct a binomial lattice for the price of an option on a stock, we need to know the actual probability that the stock will increase in value in the lattice time period.

2. Consider the following linear programming problem in canonical form:

$$\text{Maximize } Z = 2x_1 + 3x_2$$

subject to the constraints $x_i \geq 0$, $i = 1, 2$, and

$$\begin{aligned} x_1 + 2x_2 &\leq 4 \\ x_1 + x_2 &\leq 3. \end{aligned}$$

(a) Solve this problem directly by the graphical method.
(b) Express it in standard form (only inequalities are $x_i \geq 0$.)

3. Consider the system of equations

$$\begin{aligned} x_1 + x_3 &= 1 \\ x_2 - x_3 &= 2 \\ x_1 + x_2 + x_3 &= -2. \end{aligned}$$

(a) Express this system in matrix form $A\mathbf{x} = \mathbf{b}$.
(b) Write out explicitly (no matrices or vectors), the system defined by $A^T A\mathbf{x} = A^T\mathbf{b}$.

4. Consider a weighted portfolio of two bonds with face value 100: B_1 is a zero coupon bond with rate of return 6% and B_2 is a 3 year bond with annual coupons rate of 8%.

(a) Given the current risk-free interest rate is r, write formulas for the present values of B_1 and B_2.

(b) If you wanted to build a portfolio whose weighted duration is two years, what would be the system of equations you would have to solve (don't do it.) You may assume the duration of B_2 is about 2.78.

5. A portfolio consists of two assets with weights w_1 and w_2 and no short positions. Variance of the two assets are $\sigma_1 = 0.3$ and $\sigma_2 = 0.6$, and the covariance is $\sigma_{12} = -0.1$. Expected returns are $r_1 = 0.04$ and $r_2 = 0.08$.

(a) Find the expected return and variance of the portfolio in terms of w_1 and w_2.

(b) Write out the system you would have to solve to find the portfolio with minimum volatility (do not solve it.)

6. Consider the stochastic differential equation for a risky asset $S(t)$:

$$dS = \mu S\,dt + \sigma S\,dW.$$

(a) Explain what each term is.

(b) Calculate the mean and variance of dS.

Take-Home Portion

Points: 35

Due Date: Friday, March 23, 5:00 p.m. for hard copy, 11:00 p.m. for email.

Instructions: Show your work and give reasons for your answers. There is to be absolutely no consultation of any kind with anyone else other than me about the exam. If there are points of clarification or corrections, I will post them on our message board. ALL materials used in your work that have not been provided by me for this course must be explicitly credited in your write-up. Point values are indicated. You may send an email document (preferably a pdf file, but Word documents will be accepted — in all cases it must be a *single* document) or hand in hard copy at my office.

(15 pts) **1.** You are to build a portfolio from four available coupon bonds, all with face value of $\$1,000$, coupon rates of 5, 6, 7, and 8 percent and period 2, maturities 2, 4, 6, 7 years, respectively, as a weighted combination of these bonds. No short positions are allowed. The prevailing risk-free interest rate is 7%. We wish to find a portfolio whose weighted convexity is maximized subject to the constraints that the weighted duration is at most the average of the shortest and longest bond duration, and no more than 40% of the investment is in bonds of maturities greater than 5 years.

(a) Express this problem as a linear programming problem.

(b) Use Matlab to solve it.

(20 pts) **2.** You are given that the risk-free interest rate is 10%, and that a put option on a stock with volatility $\sigma = 0.4$ has 6 months from now ($t = 0$) to expiry, at which point the strike price of the option is \$50.

(a) Use Matlab tools to make a plot of the values of European and American put options at $t = 0$ as functions of stock price S, and put the payoff curve for this option on the same graph. Label the curves.

(b) Use the graphs or any other Matlab tools to estimate the value of S at which one of the curves crosses the payoff curve and the value of S where the other just touches the payoff curve.