

Midterm Exam JDEP 384H: Numerical Methods for Business

Name: _____

Score: _____

Instructions: Write out your solutions on the paper provided. Show your work and give reasons for your answers. NO notes, calculators, laptops, cell phones or other electronic equipment allowed. Point value of each problem is indicated. Work any six problems. *Clearly indicate which six you want graded.* Maximum possible score is 80. Give exact answers (like $1/3$ or $\sqrt{2}$) where calculations are expected.

(13 pts) **1.** Explain what is meant by the yield of a cash flow stream $[c_1, c_2, \dots, c_n]$ which is purchased for a price of P . Exhibit an equation that has to be solved to find the yield.

(13 pts) **2.** Express the following linear programming problem in standard form using equations and also matrix notation.

$$\text{Maximize } x_1 - x_2 + 3x_3$$

subject to the constraints

$$\begin{aligned} x_1 + 2x_3 &\geq -4 \\ x_2 - x_1 &\leq 2 \end{aligned}$$

and $x_i \geq 0, i = 1, 2, 3$.

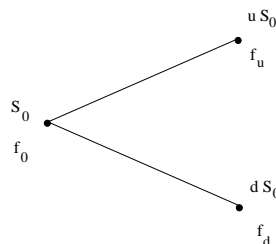
(13 pts) **3.** A portfolio consists of two assets with weights $2/3$ and $1/3$. Volatilities of the two assets are $\sigma_1 = 0.3$ and $\sigma_2 = 0.6$. Also, covariance is $\sigma_{12} = -0.1$. Compute the variance of the portfolio.

(14 pts) **4.** Explain the meaning of

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + \mathcal{O}(h^2), \quad h \rightarrow 0.$$

If you were to drop the $\mathcal{O}(h^2)$ term and use this formula to approximate $f'(x)$ with, say $h = 1/2^3$, then $h = 1/2^4$, how would you expect the error to change?

(14 pts) **5.** A single step lattice is used to describe the price f_0 of an American put at a time δt before expiry as in the picture below. Sketch how you would calculate f_0 , given that the stock price S_0 , strike price of the option K , up and down factors u, d , risk-neutral probability p , risk-free interest rate r and the formula $f_0 = e^{-r \cdot \delta t} [p f_u + (1-p) f_d]$ are all known.



(13 pts) **6.** Given two assets with volatilities $\sigma_1 = 0.3$ and $\sigma_2 = 0.6$, and covariance $\sigma_{12} = 0.1$, express the problem of finding (the covariance of) a portfolio which is a combination of these two assets (no short positions) as a mathematical problem (you do not have to solve it, but reduce it to equations not involving matrices.) How does the answer help in constructing the efficient frontier for this portfolio?

(13 pts) **7.** The vector $\mathbf{x}_T = [4; 0; 3]$ is approximated by the vector $\mathbf{x}_A = [3; -2; 1]$. Find the absolute and relative errors of these approximations.

(13 pts) **8.** Express the following system of equations in matrix form.

$$x_1 + 2x_2 - x_3 = 4$$

$$x_2 + x_3 = 1$$

$$x_1 - 3x_2 = 2$$

If you know that the right hand side is accurate to 3 significant digits and the condition number of the coefficient matrix is 10^2 , would you trust the computed answer? Why or why not?

Take-Home Portion

Points: 50

Due Date: Friday, March 10, 5:00 p.m. for hard copy, 11:00 p.m. for email.

Instructions: Show your work and give reasons for your answers. There is to be absolutely no consultation of any kind with anyone else other than me about the exam. If there are points of clarification or corrections, I will post them on our message board. ALL materials used in your work that have not been provided by me for this course must be explicitly credited in your write-up. Point values are indicated. You may send an email document (preferably a pdf file, but Word documents will be accepted) or hand in hard copy at my office.

(18 pts) **1.** You are to build a portfolio from three available coupon bonds, all with face value of \$1,000, coupon rates of 7, 6, and 8 percent and period 2, maturities 2, 4, 7 years, respectively, as a weighted combination of these bonds. (No short positions are allowed.) The prevailing risk-free interest rate is 6%. Find a portfolio whose weighted convexity is maximized subject to the constraint that the weighted duration is at most the average of the three bond durations. Describe the math problem to be solved in text and equations.

(14 pts) **2.** A portfolio consists of three assets, weights 40%, 40%, 20%, and with daily volatilities 2%, 1% and 3%, respectively. Also, the first and third have correlation -0.2 , the second and third have correlation 0 and the first and second have correlation 0.3. Find the value at risk per unit dollar over a period of 8 days at a confidence level of 96%. Describe the math problem to be solved in text and equations.

(18 pts) **3.** Use the data of the example on page 54 of the text and plot the curves of Figure 1.16 for an European call on an asset with a dividend rate of $D_0 = 0.04$ (bseurcall will help.) Next, modify the program LatticeEurCall.m to create a program LatticeAmCall.m that has the same parameters as LatticeEurCall.m plus a dividend, and computes the price of an American call option on a (constant, continuous) dividend paying asset. You need to change the lattice formulas so that the risk-free interest rate r is replaced by $r - D_0$. Make graphs as in Figure 1.16 for the American call on this asset.