(24) 1. Consider the linear system given by the following:

\[
\begin{align*}
    x_1 + x_2 + x_3 - x_4 &= 2 \\
    2x_1 + x_2 - 2x_4 &= 1 \\
    2x_1 + 2x_2 + 2x_3 - 2x_4 &= 4
\end{align*}
\]

(a) (12) Use Gauss-Jordan elimination to find the general solution to this system. Clearly specify the elementary row operations you use.

(b) (4) If we write the system as \(Ax = b\), what are the coefficient matrix \(A\) and right-hand-side vector \(b\)? What are the rank and nullity of \(A\)?

(c) (3) Express the reduced row echelon form \(R\) of the augmented matrix \(\tilde{A}\) of this system as product of elementary matrices and \(\tilde{A}\).

(d) (5) Apply the row operations used in part (a) in the same order as in (a) to a general right hand side vector \(b = (b_1, b_2, b_3)\). What is the resulting vector?
(16) 2. Let \( A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \). Find the inverse of \( A \) and use it to solve \( Ax = b \) with \( b = (2, -4, 8) \).

(12) 3. Solve the following systems for the (complex) variable \( z \). Express your answers in standard form \( (z = x + iy) \) where possible.

(a) \( z = e^{i\pi} + 2i \)

(b) \( (2 + i)z = 1 \)

(c) \( z^3 = 1 \)
(20) 4. Carry out these calculations or indicate they are impossible. You are given that \( x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \ y = \begin{bmatrix} 3 & 4 \end{bmatrix}, \ C = \begin{bmatrix} 2 & 1 + i \\ 0 & 1 \end{bmatrix}, \) and \( D = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}. \)

(a) \( yC \)

(b) \( xy \)

(c) \( x + 2x^T \)

(d) \( D^{*} \)

(e) \( C^{-1} \)

(f) \( CD \)
5. Fill in the blanks or answer True/False. Parts (a) and (b) are worth 3 points and remaining parts are worth 2 points.
(a) If $A$ is a $2 \times 2$ nonzero matrix and the system $Ax = b$ has infinitely many solutions for some $b$ then rank $A =$________ and $A$________ invertible. (Fill in “is” or “is not”.)

(b) $T((x, y)) = (x + y, 2x, 4y - x)$ is a matrix multiplication function $T_A((x, y))$, where $A =$

d) As a matrix-vector product, $x_1 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 5 \\ 8 \end{bmatrix} =$

(e) If $3 \times 3$ matrix $A$ is invertible, then the reduced row echelon form of $A$ is:

(f) Any homogeneous (right-hand-side vector 0) linear system is consistent (T/F):

(g) If $A, B$ are $2 \times 2$ matrices, then $(AB)^2 = A^2B^2$ (T/F):

(h) Every diagonal matrix is symmetric (T/F):

6. Let $A = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 1 & 3 \end{bmatrix}$.

(a) Verify the commutative law of matrix addition for these two matrices.

c) (Honors only) Give a proof of this law.