Math 314H Projectlet 1: Dimensional Analysis

DUE DATE: MONDAY, FEBRUARY 17, 2003
POINTS: 15

About Projectlets: Think of a projectlet as more than an exercise but less than a project. These are to be carried out by the smallest teams: individuals. Start by reading the assignment on your own. As with any homework assignment, it is OK to collaborate with others to the extent that you work jointly at solving a problem. It is *not* OK to copy someone else's work, and it should be clearly understood that each person is to write up his/her own work separately.

Projectlets should be typed up in some document preparation system such as WordPerfect, MSWord, Latex, etc., or *very* neatly handwritten. Like any good writing writing exercise, projectlets should have, as Aristotle advises, a beginning (introduction), middle (main body of work) and end (conclusion). In a projectlet, these parts can be as small as a paragraph. Remember that part of your grade will be based on the quality of your written work. The paper you turn in should be a mix of equations, formulas and prose. You should write your answers in such a way that it can be read and understood by anyone who knows the material for this course. Your write-up should be self-contained. Do not write it up in the succinct style of a homework exercise for your instructor.

Background: Units are often neglected in math courses, but are very important for understanding a problem. Physicists are keenly aware of this; unit checks are commonly used for confirming that an equation is correctly formed. Such equations will usually involve physical quantities, such as mass, time, length, velocity, energy, etc. Physical quantities consist of a pure number with units attached to it. For instance, we might measure velocity as $v = 20$ feet per second.

A physical law will involve physical quantities. It's reasonable to think that a physical law should be independent of units. Shouldn't Newton's law of gravity remain true whether we measure length in terms of meters or feet? We can abstract this idea of units a bit: both feet and meters are units of length. Think of length as a *dimension* of a quantity, and physical quantities as having dimensions attached to them, rather than a specific unit. Likewise, force has dimensions attached to it. Given a physical quantity q, we can try to describe dimensions in terms of other, more fundamental dimensions. For example, if in some physical problem, f represents a force, then we can say the dimensions of f are

$$[f] = \frac{\text{mass} \cdot \text{length}}{\text{time}^2} = MLT^{-2}$$

where M, L, T are the fundamental dimensions of mass, length and time, respectively. There is a kind of calculus of dimensions here. For example, if in this same physical problem an object moves a distance d, then we could calculate the dimensions of fd (work) as

$$[Fd] = [F]\,[d] = MLT^{-2} \cdot L = ML^2T^{-2}.$$

If this physical problem involves a mass m, distance d, force f and time t, we see that

$$\left[\frac{Fd}{m\,(d/t)^2}\right] = \frac{[F]\,[d]}{[m]\,[d]^2\,/\,[t]^2} = \frac{ML^2T^{-2}}{ML^2T^{-2}} = 1.$$

Thus, we would say that the quantity $Fd/(m\,(d/t)^2)$ is a *dimensionless* quantity for this problem. Are there any other dimensionless variables and how does one find them? What we do is add up all the exponents of each fundamental unit that occurs and set each sum equal to zero. To this end, it is convenient to set up a so-called *dimension matrix* of the form

One might wonder why this idea is worth anything at all.