1. Find $\Phi_{15}(x)$ and $\Phi_{20}(x)$.

2. Let $E/F$ and $F/K$ be separable (algebraic) extensions (but not necessarily finite). Prove that $E/K$ is separable.

3. Let $K \subseteq E, F \subseteq L$ be fields. Suppose $E/K$ and $F/K$ are algebraic. Prove that $EF/K$ is algebraic and that

$$EF = \{ e_1 f_1 + \cdots + e_n f_n \mid e_i \in E, f_i \in F, n \geq 1 \text{ arbitrary} \}.$$ 

4. Let $K \subseteq E, F \subseteq L$ be fields. Suppose $E/K$ and $F/K$ are separable (algebraic). Prove that $EF/K$ is separable and that $[EF : K]_s \leq [E : K]_s[F : K]_s$.

5. Let $F$ be a field of characteristic $p > 0$ and $a \in F$. Prove that $x^p - a \in F[x]$ is either irreducible or splits completely in $F[x]$. 