1. Let $n$ be a positive integer. Prove the following formula:

$$\sum_{k=1}^{n} (2k - 1) = n^2.$$ 

2. Let $p > 2$ be a prime.

   (a) Prove that $p \equiv 1 \pmod{4}$ or $p \equiv 3 \pmod{4}$.

   (b) Suppose $n > 1$ and $n \equiv 3 \pmod{4}$. Prove that $n$ is divisible by some prime $p$ with $p \equiv 3 \pmod{4}$.

   (c) Prove that there are infinitely many primes which are congruent to 3 modulo 4. (Hint: Tweak Euclid’s proof that there are infinitely many primes: suppose $p_1, \ldots, p_k$ are all the primes which are congruent to 3 modulo 4. Consider $n = p_1^2 \cdot p_2^2 \cdots p_k^2 + 2$. Show $n \equiv 3 \pmod{4}$.)

3. Find the inverse of 259 modulo 314.

4. Let $a$ and $b$ be integers (not both zero) and $m$ a positive integer. Prove that

$$\gcd(ma, mb) = m \cdot \gcd(a, b).$$

5. Prove that $\sqrt[3]{10}$ is irrational.

6. Which of the following relations are equivalence relations? Justify your answer.

   (a) Let $S$ be the set of all rectangles. For $R, T \in S$, define $R \sim T$ if and only if $R$ and $T$ have the same area.

   (b) Let $S$ be the set of all triangles. For $R, T \in S$, define $R \sim T$ if $R$ and $T$ have at least one congruent angle.

   (c) Let $S$ be the set of positive integers. For $n, m \in S$, define $n \sim m$ if $n \mid m$.

   (d) Let $S = \mathbb{Q}$, the set of rational numbers. For $a, b \in S$, define $a \sim b$ if $a - b \in \mathbb{Z}$.

7. Find $\ln(14^{3664}, 193)$. (Fermat’s Theorem is helpful here.)