[1] (10 points) Let $p$ be a prime and suppose $p \mid ab$. Prove that $p \mid a$ or $p \mid b$.

[2] (10 points) Suppose $a \equiv b \pmod{m}$. Prove that $\gcd(a, m) = \gcd(b, m)$. 
(10 points) Let $n$ be a positive integer. Prove the following formula:

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

(5 points) How many integers between 1 and 500 are relatively prime to 500?
[5] (10 points) Use the Euclidean algorithm to find $d = \gcd(176, 1122)$, and also integers $x$ and $y$ such that $d = 176x + 1122y$.

[6] (5 points) Give the definition for a relation on a set $S$ to be an *equivalence* relation.
[7] (10 points) Find the least nonnegative residue of \(2^{14004}\) modulo 29. Show all of your work.

[8] (10 points) Let \(p\) be a prime and \(a\) an integer which is not divisible by \(p\). Prove that 
\[
a^{p^3} \equiv a^{p^2} \pmod{p^3}.
\]