Each student is required to participate in a joint project with another student. The project will have both a written and an oral component. These projects can be based on interesting problems or applications that were considered in class, but which were not fully explored, or they can be chosen from the list of suggested topics below. Each pair of students will be assigned a presentation date on which they must present the oral component of their project. Additionally, the written component of the project is due on that date. There will be no extensions. Your project will count 20% of your final grade.

Written Component

Your paper should be about 5 to 10 pages long. Quality is more important than quantity. Have something to say and say it clearly and concisely. If you are presenting the results of your investigation of some journal article or textbook chapter, you should fill in the missing parts of each argument or proof and do any problems left to the reader. It would be better to go into a small part of some topic in depth and detail, rather than try to cover a large area superficially. This is your opportunity to show that you can read some mathematics on your own and then explain it in writing to your reader.

On Thursday, October 30th, you will need to tell me who your partner is and which topic you’re going to work on. I will announce which group presents on which date the following class period. (All presentations will be during the last week of classes.) The written component is due the day you do your oral presentation.

Oral Presentations

You will have 20 minutes to enlighten your colleagues about the topic you have researched. Your presentation should be clear and to the point. Choose your examples carefully to illustrate the points you want to make. In a group presentation, all in the group should have a role and all should be able to answer any questions which arise. You should rehearse your presentation in advance on some fellow students and leave some time for questions and interruptions. Class presentations always take more time than you think they will. Rehearsal will help you to better gauge how much you can accomplish. Attendance is mandatory for everyone for all oral presentations.

Some Possible Topics

Check Digit Schemes. What schemes are used for some specific types of numbers (credit cards, drivers’ licenses, passport numbers, shipping labels)? What are the advantages and disadvantages of certain schemes? Why do you think each of these particular schemes was chosen?

Mathematics and Magic. What are some of the many magicians’ tricks, especially with cards, that are built upon number theoretic facts?

The Gregorian Calendar. How does it work? How long is a complete period before the sequence of days and dates repeats? How can you find the day of the week for any date?

Mathematics and Music. What does number theory have to do with piano tuning and the “Well-tempered Klavier”? What was Bach’s contribution? What number theoretic problems arise in tuning a piano so that it can be played in any key?
Random Number Generation. How does the built-in the random number generator in your calculator or your favorite computer language or spreadsheet actually work? What are its strengths and shortcomings? How could it be improved?

Perfect Numbers and Mersenne Primes. What is the history of perfect numbers? What are some of their properties? What precisely is the connection between perfect numbers and Mersenne primes? Do odd perfect numbers exist?

Factoring Methods. We have used only trial division to factor numbers. What are some of the other techniques available? The Fermat method and the Monte-Carlo (or Pollard rho method) are both accessible to someone with the background of this course.

Primality Tests. What are some of the tests that are used to determine whether a given number is prime or composite?

Chinese Remainder Theorem. What does this say, why is it true, and how is it used? How can it be used to perform arithmetic with large numbers?

Continued Fractions. Consider these strange representations of fractions:
\[
\frac{3}{17} = \frac{1}{5 + \frac{1}{1 + \frac{1}{2}}} \quad \text{and} \quad \frac{24}{31} = \frac{1}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{3}}}}
\]

In fact every rational number can be written in this form. What about irrational numbers? Why is expressing things in this way useful?

Geometric Numbers. We all know the square numbers. What are triangular numbers and pentagonal numbers? What are some of their interesting properties?

Fibonacci Numbers. These are a source of many interesting patterns and even have a journal devoted to them. What are they and why are they important? The first few are 1, 1, 2, 3, 5, 8, 13, etc.

\(p\)-adic integers. What are \(p\)-adic integers? What are some of their properties? What can they be used for?

Resources
You may want to browse through some of the books on Number Theory in the math library. The internet also has many good sites dealing with Number Theoretic topics. Using any of the standard search engines should produce many good references. (Be careful about using websites as references — some are more reliable than others! If you want to use a website as one of your references, you should clear it with me first.) I’m happy to help you find some references if you like.