[1] (27 points) Evaluate the following indefinite integrals:

(a) \( \int \frac{\cos \sqrt{x}}{\sqrt{x}} \, dx \)

(b) \( \int x \sec^2 x \, dx \)

(c) \( \int \frac{2x^2-x-3}{x-2} \, dx \)
[2] (10 points) Use a substitution to convert the following definite integral into an integral involving trigonometric functions. Show all the steps, but do not evaluate the trig integral.

\[ \int_0^2 \sqrt{4 - x^2} \, dx \]

[3] (24 points) Decide whether the following improper integrals converge or diverge. Justify your answer as completely as you can.

(a) \( \int_5^{10} \frac{2}{\sqrt{x-5}} \, dx \)

(b) \( \int_{-\infty}^{0} xe^{2x} \, dx \)

(c) \( \int_2^{\infty} \frac{x}{(x+2)\sqrt{x+1}} \, dx \)
[4] (10 points) Let $R$ be the region bounded by $y = x^2$ and $y = x^3$. Let $S$ be the solid obtained by revolving $R$ around the line $y = -1$. Set up an integral which gives the exact volume of $S$. Do not evaluate the integral.

[5] (10 points) Consider the same region $R$ as in the preceding problem. Suppose this region forms the base of a solid and the cross-sections of the solid perpendicular to the $x$-axis are equilateral triangles. Set-up an integral which gives the volume of the solid. Do not evaluate the integral.
(a) Set up an integral which gives the area inside one leaf of the rose. Do not evaluate the integral.

(b) Set up an integral(s) which gives the area inside one leaf of the rose and also inside the circle $r = 2$. Do not evaluate the integral.