KUMUNU 2019 — Poster Abstracts

The poster session occurs 3:30–4:30 on Saturday

Presenter: Kyle Maddox
Title: Finiteness of Frobenius test exponents
Abstract: The Frobenius Test exponent of a local ring of prime characteristic is the smallest Frobenius bracket power which forces every ideal generated by a full system of parameters to agree with its (usually strictly larger) Frobenius closure. In this talk, we will discuss some cases where finiteness of this exponent is known and techniques to reduce the study of this exponent to the study of a modified version of the Hartshorne-Speiser-Lyubeznik numbers of a ring, which are related to a canonical Frobenius action on local cohomology at the maximal ideal of the ring.

Presenter: Zachary Greif
Title: Green-Lazarsfeld condition for toric edge ideals of bipartite graphs
Abstract: Previously, Ohsugi and Hibi gave a combinatorial description of bipartite graphs $G$ whose toric edge ideal $I_G$ is generated by quadrics, showing that every cycle of $G$ of length at least 6 must have a chord. This corresponds to the Green-Lazarsfeld condition $N_1$. In this paper, we investigate the higher syzygies of $I_G$ and give combinatorial descriptions of the Green-Lazarsfeld conditions $N_p$ of toric edge ideals of bipartite graphs for all $p \geq 1$. In particular, we show that $I_G$ is linearly presented (i.e. satisfies condition $N_2$) if and only if the bipartite complement of $G$ is a tree of diameter at most 3.

Presenter: Zachary Flores
Title: The weak Lefschetz for vector bundles (on $\mathbb{P}^2$)
Abstract: We discuss the Weak Lefschetz for finite length cohomology modules for vector bundles arising as sheafifications of syzygies. In particular, we show how to generalize the classical theorem that complete intersections in codimension 3 have the Weak Lefschetz, utilizing results from geometry.

Presenter: Yihui Liang
Title: Degree bounds for Gröbner bases of modules.
Abstract: Let $F$ be a non-negatively graded free module over a polynomial ring $K[x_1, \ldots, x_n]$ generated by $m$ basis elements. Let $M$ be a submodule of $F$ generated by elements with degrees bounded by $D$ and $\text{dim} F/M = r$. We prove that if $M$ is graded, the degree of the reduced Gröbner basis of $M$ for any term order is bounded by $2[1/2((Dm)^{n-r}m + D)]^{2r-1}$. If $M$ is not graded, the bound is $2[1/2(((Dm - 1)(n-r) + 1)^{n-r}m + D)]^{2r}$. This is a generalization of Dubé (1990) and Mayr-Ritscher (2013)’s bounds for ideals in a polynomial ring. In particular for inhomogeneous ideals, our bound is an improvement of Mayr-Ritscher’s bound.

Presenter: Monica Lewis
Title: The local cohomology of a parameter ideal with respect to an arbitrary ideal
Abstract: Let $S$ be a complete intersection presented as $R/J$ for $R$ a regular ring and $J$ a parameter ideal. Let $I$ be an ideal containing $J$. It is well known that the set of associated primes of $H^i_I(S)$ can be infinite, but far less is known about the set of minimal primes. In 2017, Hochster and Núñez-Betancourt showed that if $R$ has prime characteristic $p > 0$, then the finiteness of $\text{Ass} H^i_I(J)$ implies the finiteness of $\text{Min} H^{i-1}_I(S)$, raising the following question: is $\text{Ass} H^i_I(J)$ always finite? We give a positive answer when $i = 2$ but provide a counterexample when $i = 3$. The counterexample crucially requires $\text{Ass} H^2_I(S)$ to be infinite. The following question, to the best of our knowledge, is open: (under suitable hypotheses on $R$) does the finiteness of $\text{Ass} H^{i-1}_I(S)$ imply the finiteness of $\text{Ass} H^i_I(J)$? When $S$ is a domain, we give a positive answer when $i = 3$. When $S$ is locally factorial, we extend this to $i = 4$. Finally, if $R$ has prime characteristic $p > 0$ and $S$ is regular, we give a complete answer by showing that $\text{Ass} H^i_I(J)$ is finite for all $i \geq 0$. 
Presenter: Janina Letz  
Title: Smallness properties and locally complete intersections  
Abstract: A commutative noetherian ring is regular if and only if all complexes with finitely generated homology are small objects in the derived category. Similarly, a local ring is a complete intersection precisely when these complexes are proxy small. I will present some local to global principles for different smallness properties which extend this characterization to non-local rings. For some rings it is sufficient for a particular complex to be proxy small so that the ring is a locally complete intersection.

Presenter: Justin Lyle  
Title: Extremal growth of Betti numbers and rigidity of (co)homology  
Abstract: We prove several results about the vanishing of Ext or Tor for modules over a Cohen-Macaulay local ring $R$. Specifically, we provide a numerical condition under which $R$ has only trivial vanishings of Ext and Tor; we provide a sharpened version for generalized Golod rings. Using these conditions, we prove the Auslander-Reiten conjecture holds in several cases, including when $R$ is Cohen-Macaulay of codimension at most 3 or multiplicity at most 8. We also establish some new criteria for the Gorenstein property by considering Betti numbers of the canonical module.

Presenter: Taran Funk  
Title: Frobenius and homological dimensions of complexes  
Abstract: Much work has been done showing how one can use a commutative Noetherian local ring $R$ of prime characteristic, viewed as algebra over itself via the Frobenius endomorphism, as a test for flatness or projectivity of a finitely generated module $M$ over $R$. Work on this dates back to the famous results of Peskine and Szpiro and also that of Kunz. We will discuss what work has been to push this theory into arbitrary (not necessarily finitely generated) modules, and display our work done to weaken the assumptions needed to obtain these results.

Presenter: Pinches Dirnfeld  
Title: Base change along the Frobenius endomorphism and the Gorenstein property  
Abstract: Let $R$ be a local ring of positive characteristic and $X$ a complex with finitely generated nonzero homology and finite injective dimension. We prove that if derived base change of $X$ via the Frobenius endomorphism (or more generally, via a contracting endomorphism even when $R$ is of characteristic 0) has finite injective dimension then $R$ is Gorenstein.

Presenter: Marcus Robinson  
Title: BCM test ideals of mixed characteristic toric schemes  
Abstract: We provide a formula for computing the big Cohen-Macaulay test ideal in the case of monomial ideals of mixed characteristic toric schemes. Of particular interest is that this formula is consistent with the formula for the multiplier ideal in equal characteristic and test ideal in positive characteristic.