

What is a function?

August 27, 2013

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- The set of resulting output numbers is called the **range** of the function.

Function Examples

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Average low F	21.2	23.2	32.1	42.6	52.5	61.4	65.2	63.4
Record low F	-21	-20	-2	17	29	36	46	41
Record low C	-29.4	-28.9	-18.9	-8.3	-1.7	2.2	7.8	5

Table : Climate data for Bloomington IN from January to August

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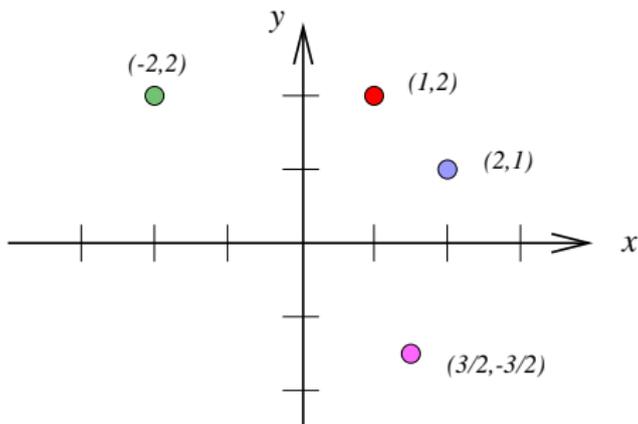
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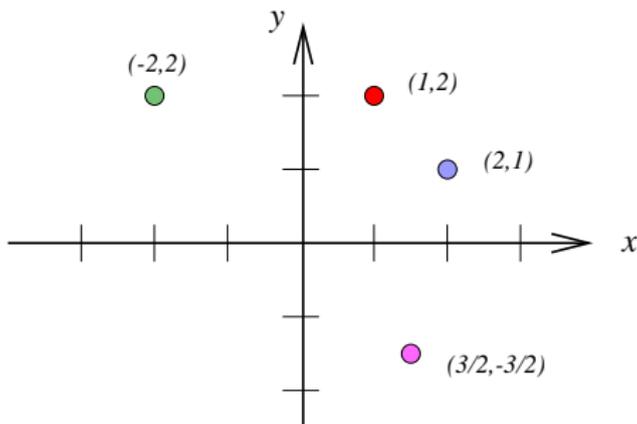
Cartesian Coordinate system

- Associate pairs of numbers with points on the plane.

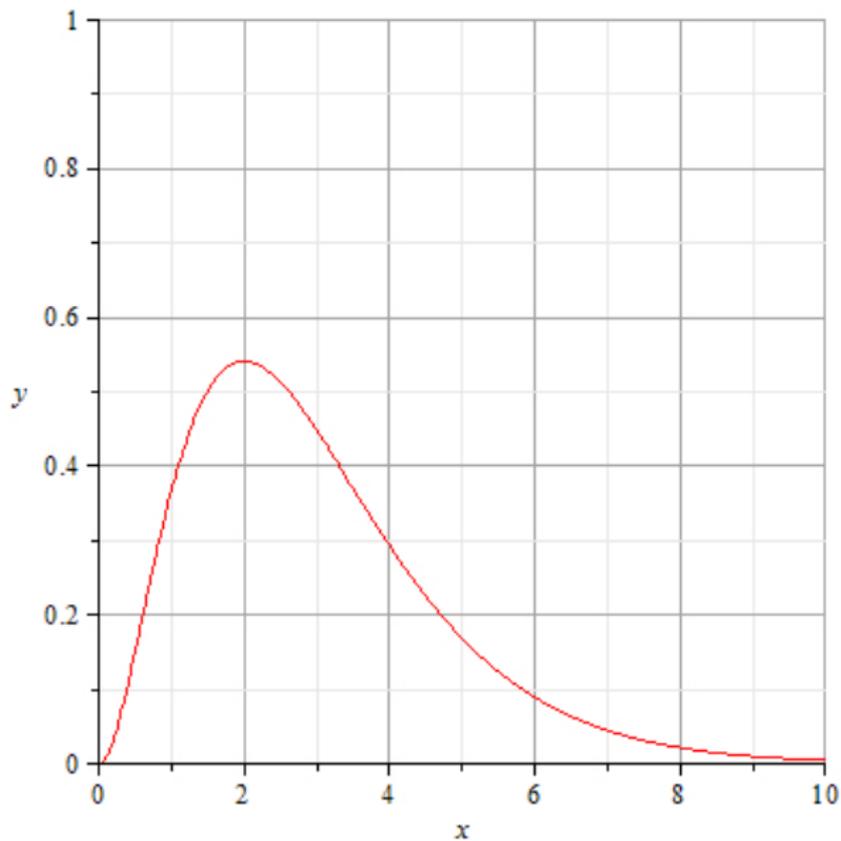


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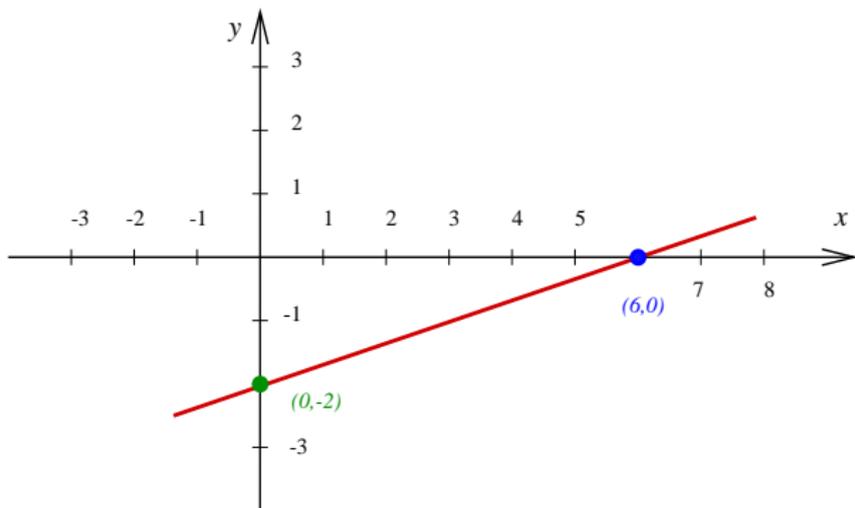
- Associate pairs of numbers with points on the plane.
- If P is the label of point (x, y) , then the numbers x and y are the **coordinates** of point P .



The rule of four: Tables, Graphs, Formulas, and Words



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- An instructor gives weekly quizzes in her mathematics course, and she seeks to find the relationship between the time required to prepare and grade a quiz and the number of students in the class. Her experience is that the average time required to prepare a quiz is 30 minutes, and the average time to grade each paper is 2 minutes.

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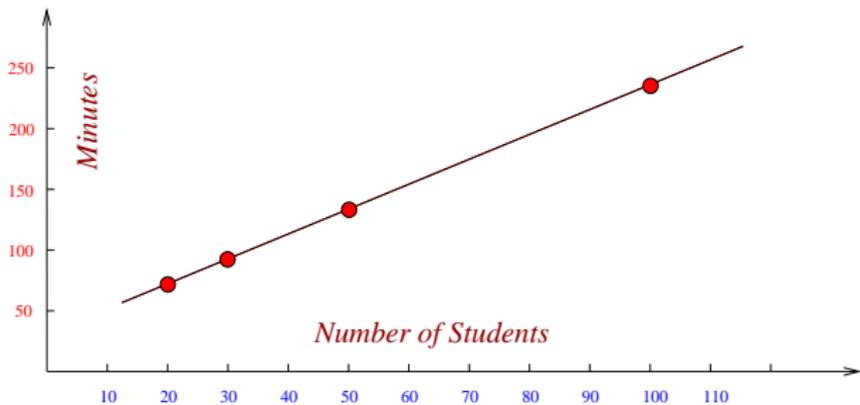
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$$y = 30 + 2x$$

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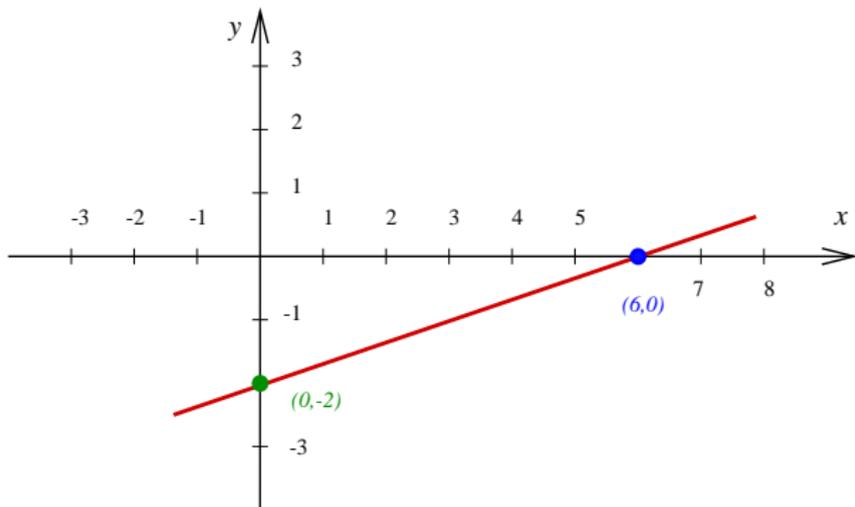
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- The graph of the function has an **intercept** where it crosses the horizontal or vertical axis.
- Horizontal intercepts are also called the **zeros** of the function.

Function notation and intercepts

Find the intercepts of the function $y = x/3 - 2$.



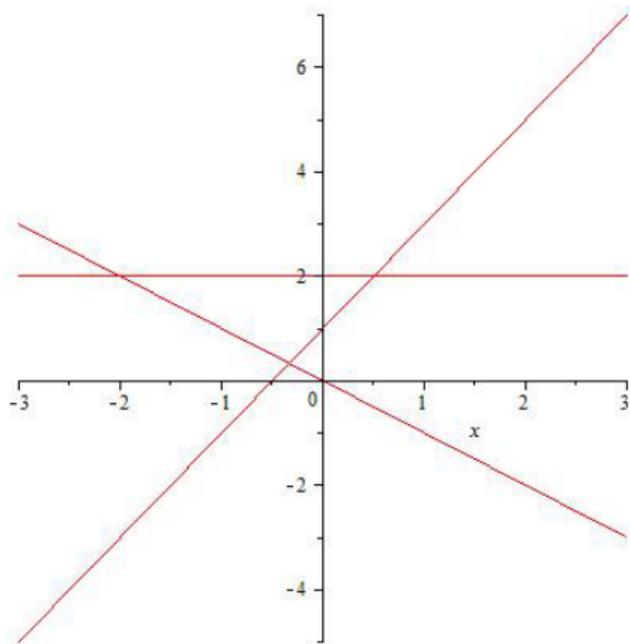
Increasing and decreasing functions

Definition

A function f is **increasing** if the value of $f(x)$ increases as x increases.

A function f is **decreasing** if the value of $f(x)$ decreases as x increases.

Increasing and decreasing functions



- For any numbers A , B and C with A and B are not both zero, the set of points

$$\{(x, y) : Ax + By = C\}$$

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- If (x_1, y_1) and (x_2, y_2) are two different points on this line, i.e. $Ax_1 + By_1 = C$ and $Ax_2 + By_2 = C$, then the line is referred as **the line through (x_1, y_1) and (x_2, y_2)** .

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- The equation $Ax + By = C$ is called the **general equation** of the line.

Slope

- The **slope** of a line is the ratio of the difference of the y coordinates to the difference of the x coordinates of any two distinct points on the line, whenever the latter difference is not zero.

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$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

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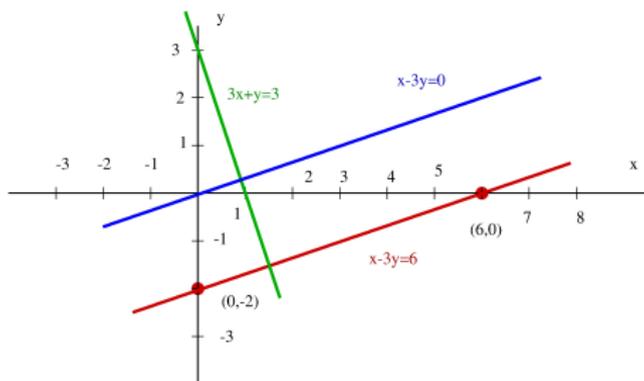
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- If $x_2 = x_1$ and $y_2 \neq y_1$, then the line is vertical and its slope is **not defined**.
- The slope m of a line does **not** depend on the choice of the points (x_1, y_1) and (x_2, y_2) .

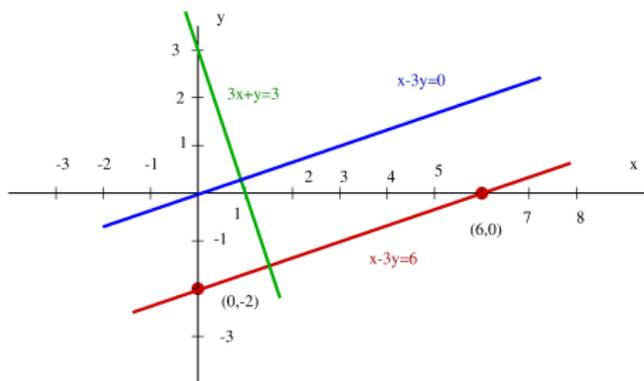
Parallel orthogonal lines

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- Two lines are parallel if: they have the same slope or if the slope is undefined for both.
- Two lines are **orthogonal** if: the slope is undefined for one of them, and the other one has slope 0; or the product of two slopes is -1 .



Linear Functions.

Definition

- A linear function has the form

$$y = f(x) = b + mx$$

Recognizing Data from a Linear Function: Values of x and y in a table could come from a linear function $y = b + mx$ if differences in y -values are constant for equal differences in x -values.

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- Its graph is a line such that:
 - m is the slope, or rate of change of y with respect to x .
 - b is the vertical intercept.

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Point-slope form of a line

- Find the equation of the line which has slope 4 and which passes through the point $(2,3)$.

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$$y - y_0 = m(x - x_0)$$