

# What is a function?

August 27, 2013

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- The set of resulting output numbers is called the **range** of the function.

# Function Examples

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Average low F	21.2	23.2	32.1	42.6	52.5	61.4	65.2	63.4
Record low F	-21	-20	-2	17	29	36	46	41
Record low C	-29.4	-28.9	-18.9	-8.3	-1.7	2.2	7.8	5

Table : Climate data for Bloomington IN from January to August

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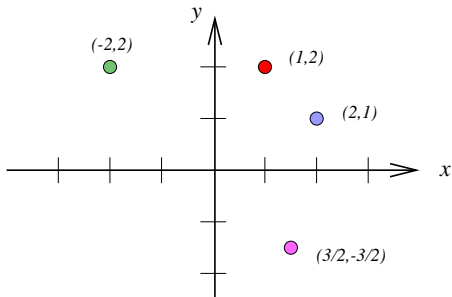
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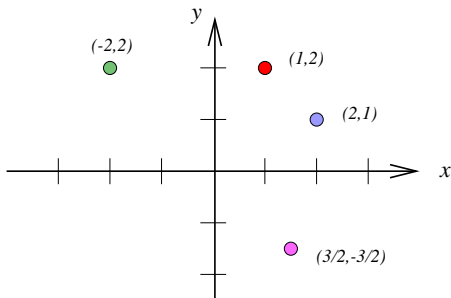
# Cartesian Coordinate system

- Associate pairs of numbers with points on the plane.

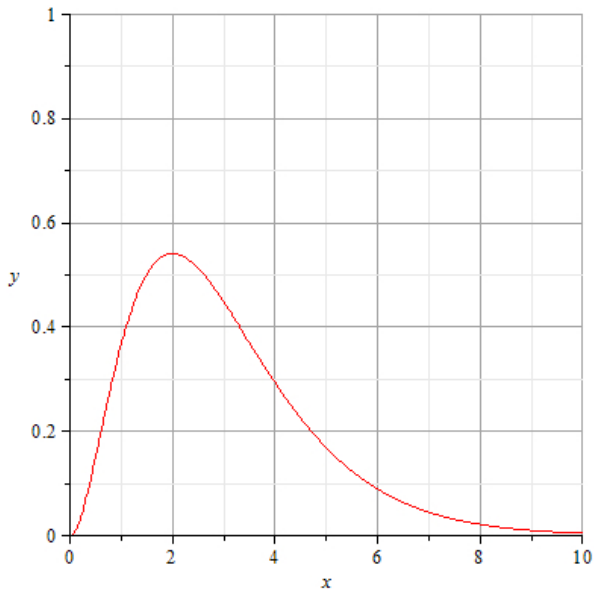


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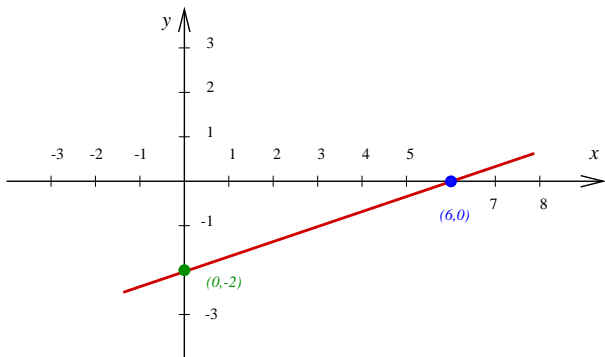
- Associate pairs of numbers with points on the plane.
- If  $P$  is the label of point  $(x, y)$ , then the numbers  $x$  and  $y$  are the **coordinates** of point  $P$ .



# The rule of four: Tables, Graphs, Formulas, and Words



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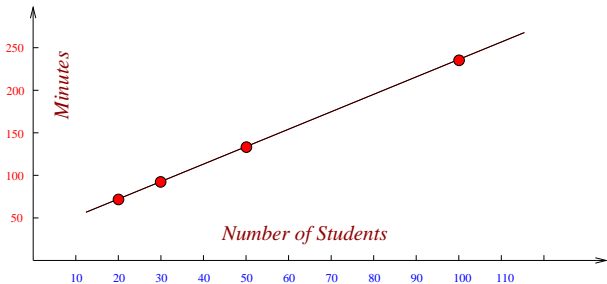
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$$y = 30 + 2x$$

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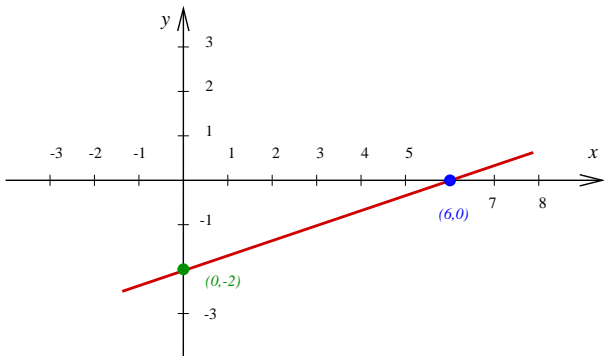
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- Horizontal intercepts are also called the **zeros** of the function.

# Function notation and intercepts

Find the intercepts of the function  $y = x/3 - 2$ .



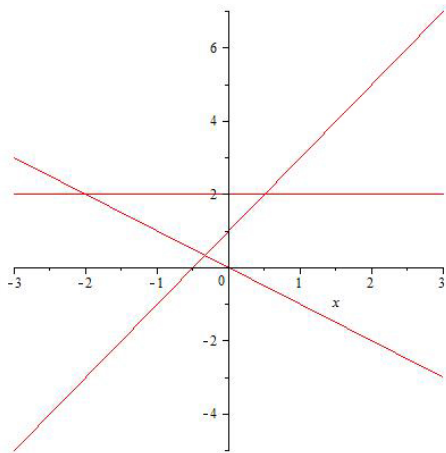
# Increasing and decreasing functions

## Definition

A function  $f$  is **increasing** if the value of  $f(x)$  increases as  $x$  increases.

A function  $f$  is **decreasing** if the value of  $f(x)$  decreases as  $x$  increases.

# Increasing and decreasing functions





- For any numbers  $A$ ,  $B$  and  $C$  with  $A$  and  $B$  are not both zero, the set of points

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- If  $(x_1, y_1)$  and  $(x_2, y_2)$  are two different points on this line, i.e.  $Ax_1 + By_1 = C$  and  $Ax_2 + By_2 = C$ , then the line is referred as **the line through  $(x_1, y_1)$  and  $(x_2, y_2)$** .

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- The equation  $Ax + By = C$  is called the **general equation** of the line.

# Slope

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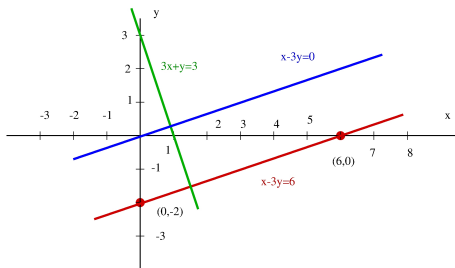
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- The slope  $m$  of a line does **not** depend on the choice of the points  $(x_1, y_1)$  and  $(x_2, y_2)$ .

# Parallel orthogonal lines

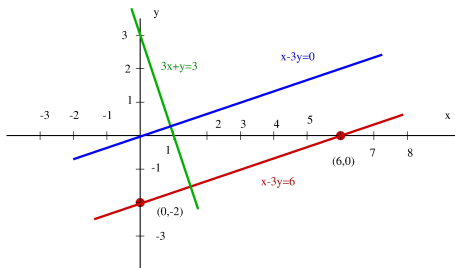
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- Two lines are **orthogonal** if: the slope is undefined for one of them, and the other one has slope 0; or the product of two slopes is  $-1$ .



# Linear Functions.

## Definition

- A linear function has the form

$$y = f(x) = b + mx$$

**Recognizing Data from a Linear Function:** Values of  $x$  and  $y$  in a table could come from a linear function  $y = b + mx$  if differences in  $y$ -values are constant for equal differences in  $x$ -values.

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- Its graph is a line such that:
  - $m$  is the slope, or rate of change of  $y$  with respect to  $x$ .
  - $b$  is the vertical intercept.

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# Point-slope form of a line

- Find the equation of the line which has slope 4 and which passes through the point  $(2,3)$ .

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$$y - y_0 = m(x - x_0)$$