

# New functions from old.

September 10, 2013

# Composite functions.

## Definition

The function  $f(g(t))$  is a function of function, or **composite function**, in which there is an **inside function** and an **outside function**.

# Examples

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- Find  $g(x - 2)$ .
- Find  $f(g(x))$  and  $g(f(x))$ .
- Find  $h(f(x) + g(2x))$ .

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- Find  $f((g(x))^2)$

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- $f(x) = \ln(3t + 1)$
- $y = 5(2x^2 + 4)^2$
- $P = (3x + 1)e^{9x+3}$

# Examples

x	0	1	2	3	4	5
f(x)	10	6	3	4	7	11
g(x)	2	3	5	8	12	15

- Find  $f(g(0))$ .



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## Definition

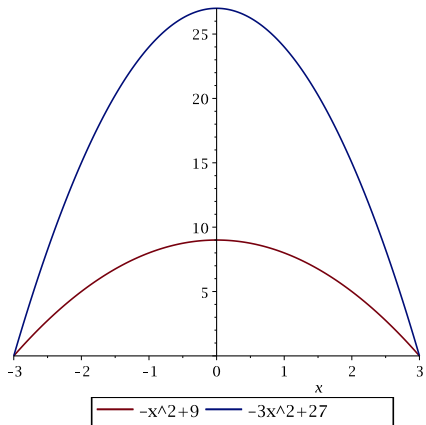
- Multiplying a function by a constant  $c$  stretches the graph vertically (if  $c > 1$ ) or shrinks the graph vertically (if  $0 < c < 1$ ).

# Stretches of Graphs

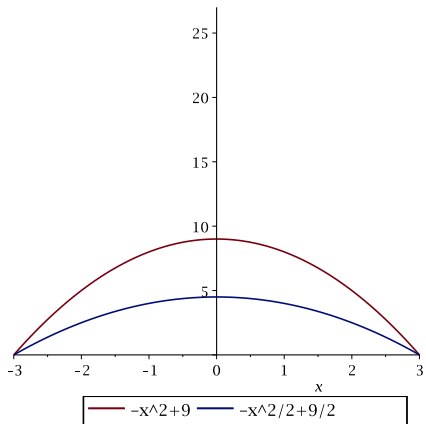
## Definition

- Multiplying a function by a constant  $c$  stretches the graph vertically (if  $c > 1$ ) or shrinks the graph vertically (if  $0 < c < 1$ ).
- A negative sign (if  $c < 0$ ) reflects the graph about  $x$ -axis in addition to shrinking or stretching.

$c > 1$

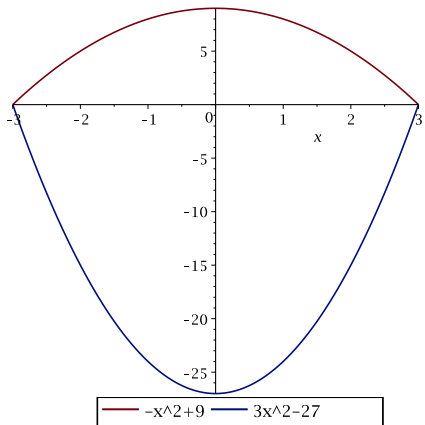


$$0 < c < 1$$

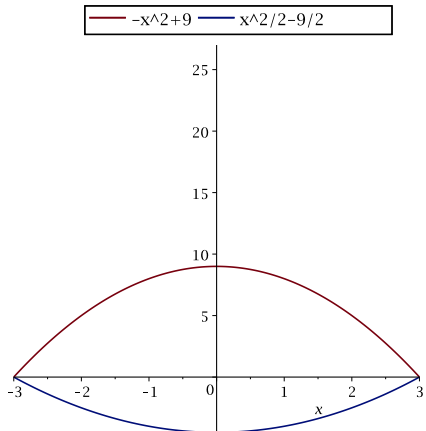




$$c < -1$$



$$-1 < c < 0$$



## Definition

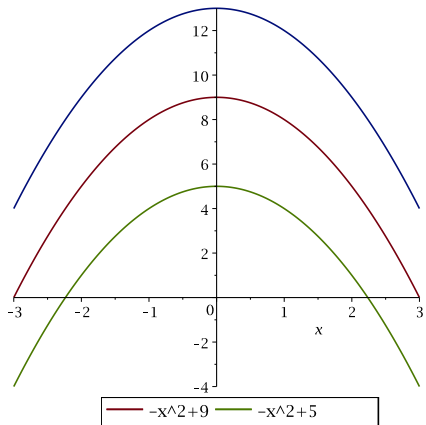
- The graph of  $y = f(x) + k$  is the graph of  $y = f(x)$  moved up  $k$  units (down if  $k$  is negative).

# Shifted graph

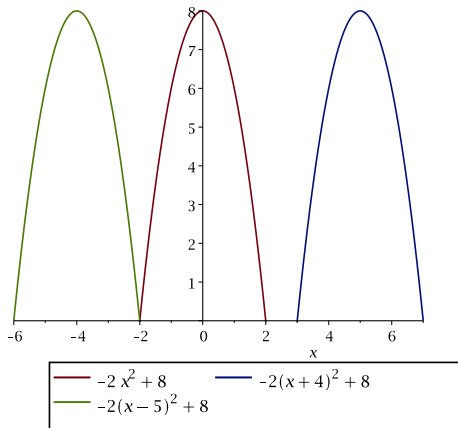
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- The graph of  $y = f(x) + k$  is the graph of  $y = f(x)$  moved up  $k$  units (down if  $k$  is negative).
- The graph of  $y = f(x - k)$  is the graph of  $y = f(x)$  moved to the right  $k$  units (to the left if  $k$  is negative).

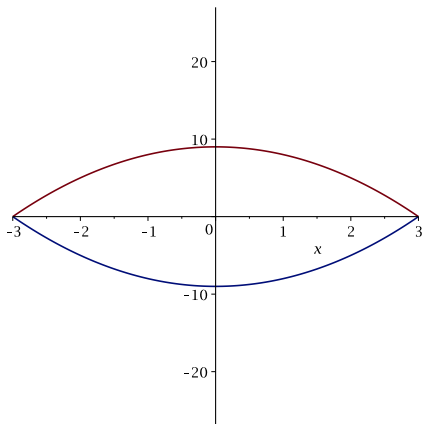
# Vertically shift



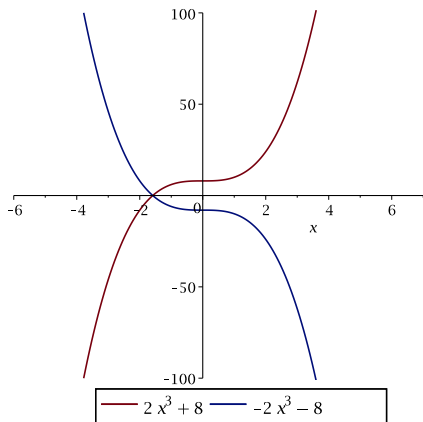
# Horizontally shift



# Reflection

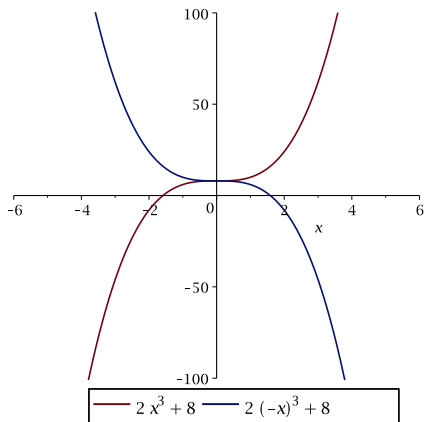


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- The graph of  $y = f(-x)$  is the graph of  $y = f(x)$  reflected about the  $y$ -axis.