

Exponential Functions

September 4, 2013

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- Find the relative change for each year between 2000 and 2003.

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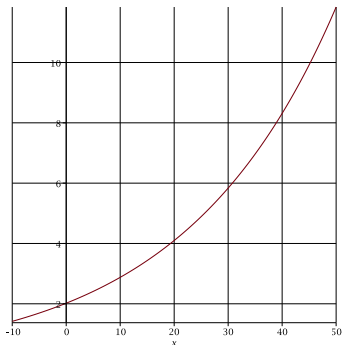
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- 1.036 represents the factor by which the population grows each year. It is called the **growth factor**.

Population Growth

Assuming that the formula holds for 50 years (since 2000).



Elimination of Drug from the Body

Problem 1. When a patient is given medication, the drug enters the bloodstream. The rate at which the drug is metabolized and eliminated depends on the particular drug. For the antibiotic ampicillin, approximately 40% of the drug eliminated every hour. A typical dose of ampicillin is 250 mg. Suppose $Q = f(t)$, where Q is the quantity of ampicillin, in mg, in the bloodstream at time t hours since the drug was given. Find several initial values of $f(t)$.

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- $f(5) = 19.4$

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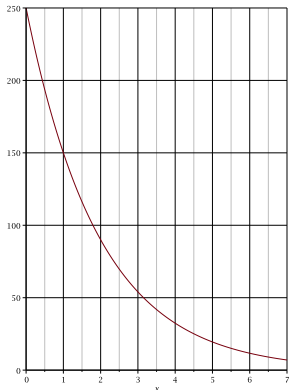
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- If $a > 1$, we have an **exponential growth**.
- If $0 < a < 1$, we have an **exponential decay**.
- $a = 1 + r$, where r is the decimal representation of the percent rate of change.

Comparison between Linear and Exponential Functions

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- An exponential function has a constant **percent** rate of change (**relative** rate of change).

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- Decreasing by 7 per minute.
- Increasing by 4% per minute.
- Decreasing by 6% per minute.

Example

Problem 3. Sales at the stores of company A increase from \$2503 millions in 1990 to \$3699 millions in 1996. Assuming the sales have been increasing exponentially, find the equation of the sale function P with respect to $t :=$ the number of years since 1990.



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$$P_0 = 2503$$



$$a^6 = 1.478$$



$$a = 1.07$$

Definition

The values of t and P in a table could come from an exponential function $P = P_0a^t$ if ratios of P values are constant for equally spaced t values.

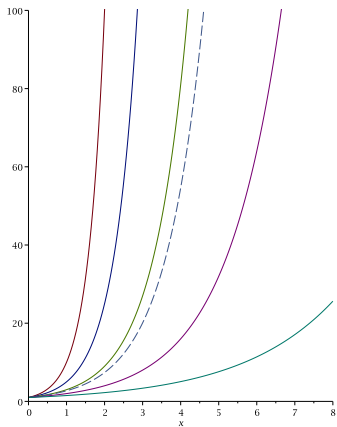
Example

x	$f(x)$
0	16
1	24
2	36
3	54
4	81

x	$g(x)$
0	14
1	20
2	24
3	29
4	35

x	$h(x)$
0	5.3
1	6.5
2	7.7
3	8.9
4	10.1

Families of exponential functions



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