Math 433/833 Spring 2009 **Project #2** Due Mon Mar 30

This project can be done using Sage or any other computer software of your choice. However, you cannot use any built-in functions that make the problem trivial.
Submit (on paper) your code with explanation—comments in the code are helpful.
You may work individually or in groups of two for this project. Each group should submit just one project report.

1. Chapter 3, Exercise 15. There is a typo in the question. It should read: show that $\mathbf{y}^{(k)} - D_k(\mathbf{d}^{(k)})$ is $\mathbf{g}(\mathbf{x}^{(k+1)})$.
2. Use Newton's Method and Broyden's Method to compute $\sqrt{2}$ to within 10^{-5}, using $x^{(0)} = 2$ for the initial point. Show the values for each iteration.
3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x_1, x_2) = (1 - x_1 + x_1^2)e^{x_2^2} + (1 - x_2 + x_2^2)e^{x_1^2}.$$

   Using $\mathbf{x}^{(0)} = (0, 0)$ as the initial point, use Newton's Method, Method of Steepest Descent, Broyden's Method, and BFGS to find the global minimizer for f. How many iterations are required for each method to get within 10^{-5}?
4. Let $\mathbf{g} : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\mathbf{g}(x, y) = (x^3 - 3xy^2, 3x^2y - y^3)$. The system $\mathbf{g}(x, y) = (1, 0)$ has three solutions $\mathbf{a}, \mathbf{b}, \mathbf{c}$; find these solutions numerically using Newton's Method. For each point of the form $\mathbf{z} = (k/100, \ell/100)$, $k, \ell \in \mathbb{Z}$, $-200 \le k, \ell \le 200$, determine which solution Newton's Method converges to when $\mathbf{z}$ is the initial point. Make three plots showing the set of points for each solution (or one plot if using color).