

This project can be done using Sage or any other computer software of your choice. However, you cannot use any built-in functions that make the problem trivial.

Submit (on paper) your code with explanation—comments in the code are helpful.

1. Find the global minimizers and maximizers, if they exist, for

$$f(x_1, x_2) = (1 - x_1 + x_1^2)e^{x_2^2} + (1 - x_2 + x_2^2)e^{x_1^2}.$$

2. The Lucas numbers L_n are defined using the Fibonacci recurrence, but with different initial values:

$$L_n = L_{n-1} + L_{n-2}, \text{ for } n \geq 1,$$

$$L_0 = 2$$

$$L_1 = 1.$$

Compute the 100th Lucas number.

3. Write a function that computes the reduced row echelon form of an $m \times n$ matrix A . Test your function against a variety of matrices; you can use the built-in function `echelon_form()` to check your answers.
4. Write a function that determines if an $n \times n$ matrix is positive definite. You may use built-in determinant or eigenvalue functions. Remember to check if the matrix is symmetric.