

Attempt all of the following problems; do not spend more than 5 minutes on any given problem. Full credit will be given to all submitted assignments.

Calculus

1. Find the maximum and minimum of $f(x) = \frac{x^2 - x}{7 - 5x - x^2 + x^3}$ on the interval $[-2, 1]$.
2. Compute the gradient $\nabla f(\mathbf{x})$ of the function $f(x_1, x_2, x_3) = 3x_1^2 + 4x_1x_2 + x_2 \sin(x_3)$.

Linear Algebra

3. Solve the following linear system of equations. If the system is consistent, write the solution in vector form.

$$\begin{cases} x & -y & +z & = & 0 \\ -x & +3y & +z & = & 5 \\ 3x & +y & +7z & = & 10 \end{cases}$$

4. For each matrix below, determine if it is invertible. If so, compute its inverse.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ -1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 0 \end{bmatrix}.$$

5. Let

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}.$$

Find all of the eigenvalues of A and bases for the corresponding eigenspaces.

6. Are the following vectors linearly independent? Prove your answer.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$