Math 850 Fall 2009 **Homework #7** Due Wed Oct 28

Warmup Problems: Section 3.4 #1,2,3,5,7. These are to practice concepts.
Other Problems: Section 3.4 #12,14,23,26. Think about some that look interesting, as time permits.
Written Problems: Do four of the following problems. Section 3.4 #6,11,21

Improved 3.4.8: The left and right pillars of an arch must have the same total height. They will be built using blocks of height 1 or 2, but blocks of height 2 may not sit on blocks of height 1. Let a_n be the number of ways to do this using a total of n blocks. Note that $a_0 = 1$, $a_1 = 0$, $a_2 = 2$, $a_3 = 2$, etc.
a) Build the OGF for $\langle a \rangle$ by showing that a_n equals the number of partitions of n using red 2s, blue 2s, green 3s, and yellow 3s in which at most one color of 3s can be used.
b) From the generating function, determine the asymptotic behavior of a_n.

Improved 3.4.19: Let a_n be the total number of 2s occurring in all partitions of n. Let b_n be the total number of nonrepeated parts over all partitions of $n-1$. Note that $a_5 = b_5 = 4$; in fact, both sequences begin $(0, 0, 1, 1, 3, 4, \ldots)$. Let $P(x)$ be the OGF for all partitions of integers. Prove the surprising fact that $a_n = b_n$ for all n by showing that both sequences have the same generating function. (*Hint*: Build each generating function, and show that each equals $x^2 P(x)/(1-x^2)$.)