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ExampleOfPhaseI

```
load simplex.sage
```

```
simplex functions loaded.
```

```
A=Matrix(ZZ,[[0,0,0,0,0],[6,1,1,-1,0],[4,2,3,0,1]]); A
```

```
[ 0  0  0  0  0]
[ 6  1  1 -1  0]
[ 4  2  3  0  1]
```

```
M=identity_matrix(ZZ,2); M
```

```
[1 0]
[0 1]
```

```
Aux=matrix(ZZ,1,2,[1]*2).stack(M); Aux
```

```
[1 1]
[1 0]
[0 1]
```

```
A=A.transpose().stack(Aux.transpose()).transpose(); A=Matrix(QQ,A); A
```

```
[ 0  0  0  0  0  1  1]
[ 6  1  1 -1  0  1  0]
[ 4  2  3  0  1  0  1]
```

```
B=[5,6] # basis of artificial variables
```

```
pivot(A,5,1,B)
```

```
pivot(A,6,2,B); A # tableau is now ready to start simplex
```

```
Pivoting tableau on col= 5 row= 1
Pivoting tableau on col= 6 row= 2
[-10  -3  -4   1  -1   0   0]
[  6   1   1  -1   0   1   0]
[  4   2   3   0   1   0   1]
```

```
pivot(A,1,2,B); A
```

Pivoting tableau on col= 1 row= 2

$$\begin{array}{l} [-4 \quad 0 \quad 1/2 \quad 1 \quad 1/2 \quad 0 \quad 3/2] \\ [4 \quad 0 \quad -1/2 \quad -1 \quad -1/2 \quad 1 \quad -1/2] \\ [2 \quad 1 \quad 3/2 \quad 0 \quad 1/2 \quad 0 \quad 1/2] \end{array}$$

```
# We have obtained optimal solution.
```

```
A=Matrix(ZZ,[[0,0,0,0],[4,2,1,1],[2,1,1,2]]); A
```

```
[0 0 0 0]
[4 2 1 1]
[2 1 1 2]
```

```
M=identity_matrix(ZZ,2)
```

```
Aux=matrix(ZZ,1,2,[1]*2).stack(M)
```

```
A=A.transpose().stack(Aux.transpose()).transpose(); A=Matrix(QQ,A); A
```

```
[0 0 0 0 1 1]
[4 2 1 1 1 0]
[2 1 1 2 0 1]
```

```
B=[4,5]
```

```
pivot(A,4,1,B)
```

```
pivot(A,5,2,B); A # tableau is now ready to start simplex
```

```
Pivoting tableau on col= 4 row= 1
Pivoting tableau on col= 5 row= 2
[-6 -3 -2 -3 0 0]
[ 4  2  1  1  1  0]
[ 2  1  1  2  0  1]
```

```
pivot(A,1,2,B); A
```

```
Pivoting tableau on col= 1 row= 2
[ 0  0  1  3  0  3]
[ 0  0 -1 -3  1 -2]
[ 2  1  1  2  0  1]
```

```
# We have obtained an optimal solution with cost = 0, but y1 is still in the basis!
```

```
pivot(A,2,1,B); A
```

```
Pivoting tableau on col= 2 row= 1
[ 0  0  0  0  1  1]
[ 0  0  1  3 -1  2]
```

$[2 \quad 1 \quad 0 \quad -1 \quad 1 \quad -1]$

We now have a BFS and a basis for Phase II.