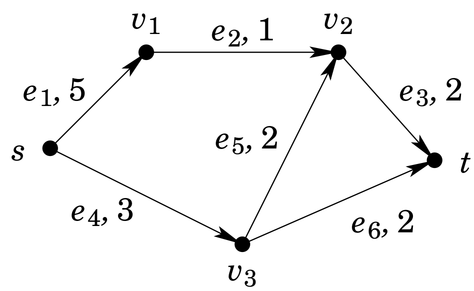
Math 432/832 Fall 2008 **Homework #9** Due Wed Dec 10

Do 3 of the following 4 problems.

1. Starting from the dual feasible vector $\mathbf{p} = [0, 1/3, 0]$, use the primal-dual algorithm to find an optimal solution to the problem

$$\begin{array}{rrrrrcl} \min & 4x_1 & +x_2 & +3x_3 & +x_4 & & \\ \text{subject to} & x_1 & +x_2 & +x_3 & & = & 3 \\ & x_1 & & +x_3 & +3x_4 & = & 2 \\ & & x_2 & +x_3 & +x_4 & = & 2 \\ & x_1, & x_2, & x_3, & x_4 & \geq & 0 \end{array}$$

2. Find a maximum flow in the graph below using both the network simplex algorithm and the primal-dual algorithm (*i.e.*, the Ford-Fulkerson algorithm).



3. *A simple cutting plane.*

   (a) Consider the following integer program:

$$\begin{array}{lrrrcl} \text{P:} & \min & x_1 & +x_2 & & \\ & \text{subject to} & 4x_1 & +x_2 & \geq & 3 \\ & & 2x_1 & -2x_2 & \geq & 3 \\ & & x_1, & x_2 & \geq & 0 \\ & & x_1, & x_2 & & \text{integer} \end{array}$$

   Solve the linear programming relaxation of P, obtaining an optimal solution $\mathbf{x}^*$ with cost z^*. If z^* is not an integer, add the cut $\mathbf{c}'\mathbf{x} \geq \lceil z^* \rceil$ to the integer program. Iterate until an optimal solution with integral cost is obtained. Does the corresponding solution have integral components?

   (b) Does the method in the previous problem of iteratively adding cuts of the form $c'x \geq \lceil z^* \rceil$ always result in an integer program whose cost is the same as the linear programming relaxation? Here, no assumption is made about the integrality of the optimal solution, just its cost.

4. Solve the following integer program using the branch-and-bound method.

$$\begin{array}{rrrcl} \min & x_1 & +x_2 & & \\ \text{subject to} & x_1 & +2x_2 & \geq & 2 \\ & -x_1 & +4x_2 & \leq & 3 \\ & x_1, & x_2 & \geq & 0, \text{ integer} \end{array}$$