Math 432/832 Fall 2008 **Homework #7** Due Wed Nov 12

Do 3 of the following 4 problems.
Other interesting problems: 5.2,5.3,5.4,5.9

1. Use the dual simplex method to find an optimal solution to the following linear program.

$$\begin{array}{rrrrrcr} \text{min} & 7x_1 & +x_2 & +3x_3 & +x_4 & & \\ \text{subject to} & 2x_1 & -3x_2 & -x_3 & +x_4 & \geq & 8 \\ & 6x_1 & +x_2 & +2x_3 & -2x_4 & \geq & 12 \\ & -x_1 & +x_2 & +x_3 & +x_4 & \geq & 10 \\ & x_1, & x_2, & x_3, & x_4 & \geq & 0 \end{array}$$

   Use the optimal tableau to determine an optimal solution $\mathbf{p}$ to the dual. Form the dual problem and verify that $\mathbf{p}$ is feasible.

2. Consider the following LP:

$$\begin{array}{rrrrcr} \text{min} & 2x_1 & +x_2 & +x_3 & & \\ \text{subject to} & x_1 & -x_2 & +x_3 & \leq & 1 \\ & -2x_1 & -x_2 & & \leq & -1 \\ & -x_1 & +2x_2 & -2x_3 & \leq & -2 \\ & x_1, & x_2, & x_3 & \geq & 0 \end{array}$$

   After solving, the following optimal tableau is obtained:

$$\left[\begin{array}{rrrrrrrr} & & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ & -3 & 0 & 0 & 0 & 5 & 2 & 3 \\ x_2 = & 1 & 0 & 1 & 0 & -4 & -1 & -2 \\ x_1 = & 0 & 1 & 0 & 0 & 2 & 0 & 1 \\ x_3 = & 2 & 0 & 0 & 1 & -5 & -1 & -3 \end{array}\right]$$

   Let $B = [A_2, A_1, A_3]$ be the optimal basis and $\mathbf{x}^*$ the corresponding optimal basic feasible solution. For each change listed below made to the original linear program, answer the following questions: Is $\mathbf{x}^*$ an optimal solution? Is B an optimal basis? What is the new optimal solution? (Note: Each question below is asking about changes to the original linear program.)

   (a) The constraint $3x_1 + 2x_2 - x_3 \leq 2$ is added.

   (b) The constraint $x_1 + x_2 = 5$ is added.

   (c) The original cost c_2 of x_2 is changed to 2.

   (d) b_2 on the right side of the original linear program is changed to 1.

3. Exercise 5.5

4. Exercise 5.7