Math 432/832 Fall 2008 **Homework #6** Due ~~Mon Oct 27~~ Wed Oct 29

Do 4 of the following 5 problems.
Other interesting problems: 4.17,4.22,4.23,4.24,4.25,4.27,4.28,4.29,4.30,4.31

1. Alice and Bob are playing rock-paper-scissors. If a player wins one round of play, his or her payoff is 1 dollar, regardless of how that person won the round. Compute the optimal strategies for both Alice and Bob. Who can expect long-term gain while playing this game? Interpret your results; are they all that surprising?

2. Alice, being bored, decides to shake things up a bit. She declares that she is adding dynamite to her arsenal. Dynamite, she explains, will beat both rock and paper (for reasons that should be obvious); however, scissors will beat dynamite since they can cut the fuse. Bob is a little bitter that he can't use dynamite in his play, so he makes a deal: Alice can use dynamite, and if she wins a round with it her payoff will be 1 dollar. If Bob manages to beat Alice when she plays dynamite, his payoff will be M dollars (Bob is thinking fast, and hasn't quite decided what M ought to be yet). Aside from these modifications, the payoff scheme is as before.

   (a) Bob decides that M ought to be 1, since all the other payoffs in the game are equal to 1 dollar. Does setting $M = 1$ make the game fair?

   (b) Determine optimal strategies and the corresponding optimal payoffs for Alice using several values of M. You may, for example, try values of M in the interval $[-3, 3]$, such as $-3, -2\frac{2}{3}, -2\frac{1}{3}, \ldots, 2\frac{2}{3}, 3$. Based on your findings, are there any values of M for which Bob can expect long-term gain from this game?

3. Exercise 4.26

4. Solve the following LP using the dual simplex algorithm.

$$\begin{array}{rrrrcr}
\min & 2x_1 & +x_2 & +x_3 & & \\
\text{subject to} & x_1 & -x_2 & +x_3 & \leq & 1 \\
 & -2x_1 & -x_2 & & \leq & -1 \\
 & -x_1 & +2x_2 & -2x_3 & \leq & -2 \\
 & x_1, & x_2, & x_3 & \geq & 0
\end{array}$$

5. Provide a short proof that the following linear program is infeasible using Farkas' lemma.

$$\begin{array}{rrrcr}
\min & -3x_1 & -x_2 & & \\
\text{subject to} & x_1 & -x_2 & \leq & -1 \\
 & -x_1 & -x_2 & \leq & -3 \\
 & 2x_1 & +x_2 & \leq & 2 \\
 & x_1, & x_2 & \geq & 0
\end{array}$$