Math 432/832 Fall 2008 **Homework #1**

Do 4 of the following 5 problems. Due Wed Sept 3.
Other interesting problems: 1.1, 1.2, 1.12, 1.13

1. State in general form ($\min c'\mathbf{x}$, $A\mathbf{x} \geq \mathbf{b}$):

$$\begin{array}{ll} \text{maximize} & x_1 + 3x_2 - 4x_3 \\ \text{subject to} & 6x_1 + x_2 - 4x_3 \leq -5 \\ & -x_1 - 3x_2 + 4x_3 \geq -9 \\ & x_1 + 2x_2 - 3x_4 = 6 \\ & x_1, x_3 \geq 0 \end{array}$$

2. Solve geometrically the problem

$$\begin{array}{ll} \text{maximize} & 2x_1 + 3x_2 \\ \text{subject to} & 3x_1 + 8x_2 \leq 12 \\ & x_1 + x_2 \leq 2 \\ & 8x_1 + x_2 \leq 14 \\ & x_1, x_2 \geq 0 \end{array}$$

3. Exercise 1.14ac.
4. Consider the following two optimization problems:

$$\begin{array}{llll} \text{P1:} & \min & \dfrac{\mathbf{c}'\mathbf{x} + d}{\mathbf{e}'\mathbf{x} + f} & \\ & \text{subject to} & A\mathbf{x} & = \mathbf{b} \\ & & \mathbf{e}'\mathbf{x} + f & > 0 \end{array}$$

$$\begin{array}{llll} \text{P2:} & \min & \mathbf{c}'\mathbf{y} + dz & \\ & \text{subject to} & A\mathbf{y} - \mathbf{b}z & = \mathbf{0} \\ & & \mathbf{e}'\mathbf{y} + fz & = 1 \\ & & z & \geq \mathbf{0} \end{array}$$

where $\mathbf{x}, \mathbf{y}, \mathbf{c}, \mathbf{e} \in \mathbb{R}^n$, A is an $m \times n$ matrix, $\mathbf{b} \in \mathbb{R}^m$, and $z \in \mathbb{R}$. Under the assumption that the feasibility sets are non-empty, show P1 and P2 are equivalent using the steps below. (Caution: Prove explicitly that a solution is feasible and has a given objective function value.)

   (a) Let $\mathbf{x}$ be a feasible solution of P1. Then construct a feasible solution $(\mathbf{y}, z)$ of P2 that has the same objective function value.

   (b) Let $(\mathbf{y}, z)$ be a feasible solution of P2. Then for $z \neq 0$, construct a feasible solution $\mathbf{x}$ of P1 that has the same objective function value. (Optional: If $z = 0$, what can we do?)

5. Prove the following:

   (a) The intersection of any collection of convex sets is convex.

   (b) The set of feasible points of a linear program is convex.