FarkasExampleFromTest1

```
load simplex.sage
```

```
simplex functions loaded.
```

This example is from Question 4 on Test 1. After forming the dual used in Farkas' lemma, we need to convert to standard form. For each free variable p_i, we write $p_i = p_i^+ - p_i^-$. We also add slack variables to convert inequalities to equalities.

```
A=Matrix(QQ,[[0,2,-2,4,-4,-2,2,0,0,0],[0,1,-1,1,-1,0,0,-1,0,0]
,[0,1,-1,-1,1,1,-1,0,-1,0], [0,-2,2,-4,4,1,-1,0,0,-1] ]);
c=A[0,1:];
B=[7,8,9]; A
```

```
[ 0  2 -2  4 -4 -2  2  0  0  0]
[ 0  1 -1  1 -1  0  0 -1  0  0]
[ 0  1 -1 -1  1  1 -1  0 -1  0]
[ 0 -2  2 -4  4  1 -1  0  0 -1]
```

```
# We need to convert the columns corresponding to slack
variables into columns of the identity matrix.
prepare_tableau(A,B); A
```

```
[ 0  2 -2  4 -4 -2  2  0  0  0]
[ 0 -1  1 -1  1  0  0  1  0  0]
[ 0 -1  1  1 -1 -1  1  0  1  0]
[ 0  2 -2  4 -4 -1  1  0  0  1]
```

```
pivot(A,B,2,1)
```

```
Pivoting tableau on col= 2 row= 1
[ 0  0  0  2 -2 -2  2  2  0  0]
[ 0 -1  1 -1  1  0  0  1  0  0]
[ 0  0  0  2 -2 -1  1 -1  1  0]
[ 0  0  0  2 -2 -1  1  2  0  1]
```

```
pivot(A,B,3,1)
```

```
Pivoting tableau on col= 3 row= 1
[ 0 -2  2  0  0 -2  2  4  0  0]
[ 0  1 -1  1 -1  0  0 -1  0  0]
[ 0 -2  2  0  0 -1  1  1  1  0]
[ 0 -2  2  0  0 -1  1  4  0  1]
```

```
pivot(A,B,1,1)
```

```
Pivoting tableau on col= 1 row= 1
[ 0  0  0  2 -2 -2  2  2  0  0]
```

```
[ 0  1 -1  1 -1  0  0 -1  0  0]
[ 0  0  0  2 -2 -1  1 -1  1  0]
[ 0  0  0  2 -2 -1  1  2  0  1]
```

Since every entry of the fourth col is nonpositive, the optimal cost is negative infinity. We can construct a sequence of feasible points with cost tending to negative infinity by moving in the 4th basic direction.

```
B
```

```
[1, 8, 9]
```

```
x=basic_solution(A,B); x
```

```
(0, 0, 0, 0, 0, 0, 0, 0, 0)
```

Remember that the jth basic direction is determined by $d_j = 1$, $d_i = 0$ for all other nonbasic i, and $d_B = -B^{-1}A_j$.

```
d=basic_feasible_direction(A,B,4); d
```

```
(1, 0, 0, 1, 0, 0, 0, 2, 2)
```

```
for t in xrange(5):
 print "t=",t,"x+t*d=",x+t*d,"cost=",(c*(x+t*d))[0]
```

```
t= 0 x+t*d= (0, 0, 0, 0, 0, 0, 0, 0, 0) cost= 0
t= 1 x+t*d= (1, 0, 0, 1, 0, 0, 0, 2, 2) cost= -2
t= 2 x+t*d= (2, 0, 0, 2, 0, 0, 0, 4, 4) cost= -4
t= 3 x+t*d= (3, 0, 0, 3, 0, 0, 0, 6, 6) cost= -6
t= 4 x+t*d= (4, 0, 0, 4, 0, 0, 0, 8, 8) cost= -8
```

Thus, picking $\theta = 1$ gives a feasible solution $(1, 0, 0, 1, 0, 0, 0, 2, 2)$ with cost -2. Converting back to the original dual (before putting in standard form), we have $\mathbf{p} = (1, -1, 0)$ with cost -2.