Math 450 Combinatorics Homework #10 Due Wednesday, November 7

Do 5 of the following problems.

1. *Flags on flagpoles*. Suppose that you have n different flags that you want to put on k distinguishable flagpoles. For a fixed k, let h_n denote the number of ways of putting the n flags on the k flagpoles. Find the exponential generating function for h_n when k is 1 and 2, and then generalize for arbitrary k.

2. Prove that the number of partitions of n into *distinct odd* parts is equal to the number of self-conjugate partitions of n.

3. Show that the number of ways of writing n as a sum of positive integers (one term is allowed) is 2^{n-1} for $n \geq 1$. Note that order of the summands matters. For instance, $1+2+3$, $2+1+3$, and $3+3$ are different ways of writing $n = 6$ as a sum of positive integers.

4. Consider the generating function $\phi_k(t) = \sum_{n=0}^{\infty} S(n,k)t^n$ for the Stirling numbers of the second kind, where k is fixed.

   (a) Find a recurrence expressing ϕ_k in terms of ϕ_{k-1}.

   (b) Deduce an explicit expression for ϕ_k by iterating the recurrence from part (a).

5. 8.6.19

6. 8.6.20