Math 450 Combinatorics Homework #8 Due Friday, October 26

Do 5 of the following problems.

1. 7.8.42 b,c

2. 7.8.44

3. 7.8.48

4. (a) Prove that if

$$a_n = \sum_{k=0}^{n} \binom{n}{k} b_{n-k}, \quad \text{for all } n \geq 0, \tag{1}$$

then

$$g_a^{(e)}(x) = e^x g_b^{(e)}(x),$$

where $g_a^{(e)}(x)$ is the exponential generating function for a_n and $g_b^{(e)}(x)$ is the exponential generating function for b_n.

   (b) Use part 4a to prove that the exponential generating function for D_n is

$$g^{(e)}(x) = \sum_{n=0}^{\infty} D_n \frac{x^n}{n!} = \frac{e^{-x}}{1-x},$$

where D_n is the number of derangements of n symbols.

5. Let $f(n, \ell)$ be the number of lattice paths from $(0, 0)$ to $(n, n + \ell)$ that never pass *above* the line $y = x + \ell$ in the plane. (Remember that lattice paths take steps of length 1 either horizontally to the right or vertically up.) Prove that

$$f(n, \ell) = \frac{\ell + 1}{n + \ell + 1} \binom{2n + \ell}{n}.$$

6. 8.6.1 or 8.6.2 (you only are required to do 1).