Math 450 Combinatorics Homework #2 Due Wed Sept 12, 2007

Do 5 out of the following 6 problems.

1. We return to the derangements problem: Given n letters and n addressed envelopes, in how many ways can the letters be placed in the envelopes so that no letter is in the correct envelope? Let $d(n)$ denote the number of ways.

   (a) Construct a recurrence formula for $d(n)$ in terms of $d(n-1)$ and $d(n-2)$.

   (b) Using induction, prove that
   $$d(n) = n!\sum_{i=0}^{n} \frac{(-1)^i}{i!}.$$

2. Let K_{mn} be an $m \times n$ chessboard, with m and n odd. Suppose that the squares of K_{mn} are alternately colored black and white, with the corners colored white. Let $\check{K}_{mn}$ be formed by removing any white square. Prove that $\check{K}_{mn}$ can be tiled with dominoes. (*Hint*: Try induction on $m + n$.)

3. Prove that the number of ways to arrange k nonattacking indistinguishable rooks on an $n \times n$ chessboard is $\binom{n}{k}P(n, k)$, where $k \leq n$. Then do 3.6.27.

4. 3.6.28. (*Hint*: Create a bijection a set that is more easily counted. Think about giving directions.)

5. 3.6.48

6. 3.6.38