Math 450 Combinatorics Homework #1 Due Wed Sept 5, 2007

Do 5 out of the following 6 problems.

1. Given n letters and n addressed envelopes, in how many ways can the letters be placed in the envelopes so that no letter is in the correct envelope? Solve this problem for the special cases $n = 3, 4, 5$. Compare to $n!$ in each case. (Can you say something about the ratio to $n!$?)

2. Let K_{mn} be an $m \times n$ chessboard. Is there a way to move a knight around the board so that it lands on every square exactly once? (The starting square can be any square that you want.) Solve this problem for 3×3, 3×4, and 3×5 chessboards. Find an infinite family of chessboards for which no knight's tour exists.

3. Let K_{nn} be an $n \times n$ chessboard, and let $\hat{K}_{nn}$ be formed from K_{nn} by removing the lower left corner square. Prove that $\hat{K}_{nn}$ cannot be tiled by L-shaped trominoes () if n is odd and n is a multiple of 3. Using induction, prove that $\hat{K}_{nn}$ can be tiled by L-shaped trominoes if n is a positive power of 2.

   Extra Credit: Show that $\hat{K}_{nn}$ can be tiled by L-shaped trominoes if n is at least 2 and n is not congruent to 0 modulo 3. Show that $\hat{K}_{nn}$ cannot be tiled by L-shaped trominoes if n is congruent to 0 modulo 3.

4. 1.8.26 and 1.8.27

5. 1.8.35

6. 3.6.7