Math 314 Fall 2007 **Quiz #8** Name: Solutions

Show all of your work; answers without work may receive no credit. Make sure that you answer each question completely.

Let

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}.$$

1. (10 pts) Find all of the eigenvalues of A and bases for the corresponding eigenspaces.

   Solution. The characteristic polynomial of A is

   $$\det(A - \lambda I) = \det \begin{bmatrix} 1-\lambda & 3 \\ -2 & 6-\lambda \end{bmatrix} = (1-\lambda)(6-\lambda) + 6 = \lambda^2 - 7\lambda + 12 = (\lambda - 3)(\lambda - 4).$$

   The solutions of $\det(A - \lambda I) = 0$ are the eigenvalues of A. Hence, the eigenvalues are $\lambda_1 = 3$ and $\lambda_2 = 4$.

   To find bases for the corresponding eigenspaces, we find a basis for the nullspace of $\det(A - \lambda I)$ for each eigenvalue λ. For $\lambda_1 = 3$,

   $$A - 3I = \begin{bmatrix} -2 & 3 \\ -2 & 3 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -3/2 \\ 0 & 0 \end{bmatrix}, \quad \text{so} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} x_2, \quad \text{so } E_3 = \operatorname{span}\left\{ \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} \right\}.$$

   For $\lambda_2 = 4$,

   $$A - 4I = \begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \quad \text{so} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} x_2, \quad \text{so } E_4 = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

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