Math 314 Fall 2007 **Quiz #6** Name: Solutions

Show all of your work; answers without work may receive no credit. Make sure that you answer each question completely.

Suppose that we play a small travel-version of Monopoly that only has 2 squares: Go and Boardwalk. Each turn consists of the following: If you are on Go, flip a fair coin: if heads, go to Boardwalk; if tails, stay on Go. If you are on Boardwalk, then always move to Go.

1. (3 pts) Write down the transition matrix for the Markov chain representing the position of the token. Remember that the transition matrix P has entries p_{ij}, which is the probability of transitioning *to* state i *from* state j. Let Go be state 1 and Boardwalk state 2.

   Solution.
$$P = \begin{bmatrix} 1/2 & 1 \\ 1/2 & 0 \end{bmatrix}.$$
□

2. (7 pts) Find the stationary distribution (or steady-state vector, as the book calls it) for this Markov chain.

   Solution. We wish to solve for $P\mathbf{x} = \mathbf{x}$, which is equivalent to $(P - I)\mathbf{x} = \mathbf{0}$. In other words, we wish to find a probability vector $\mathbf{x}$ that is also in the null space of $P - I$.
$$P - I = \begin{bmatrix} -1/2 & 1 \\ 1/2 & -1 \end{bmatrix} \xrightarrow[RREF]{} \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}.$$
   Hence, the null space consists of all vectors of the form
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} x_2.$$
   To obtain a probability vector, we need $x_1, x_2 \geq 0$ and $x_1 + x_2 = 1$. Thus, we choose $x_2 = 1/3$ and so $x_1 = 2x_2 = 2/3$. Thus, the stationary distribution is
$$\begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}.$$
□