

Show all of your work; answers without work may receive no credit. Make sure that you answer each question completely.

1. If A is a 2×5 matrix, what are the possible values of $\text{nullity}(A)$? Explain your answer!

Solution. We know that $\text{rank}(A) + \text{nullity}(A) = (\# \text{ of cols of } A) = 5$. However, the rank of A is at most 2 since the dimension of the row space can be at most 2. Since the rank of A can be 0, 1, or 2, the only possible values for $\text{nullity}(A)$ are 3, 4, and 5. \square

2. For each possible value k in Question 1, construct a 2×5 matrix A with $\text{nullity}(A) = k$.

Solution. As noted above, this is equivalent to constructing matrices with ranks 0, 1, and 2. Let

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A'' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Since these matrices are in reduced row echelon form, it is easy to compute their rank by counting the number of nonzero rows: $\text{rank}(A) = 0$, $\text{rank}(A') = 1$, and $\text{rank}(A'') = 2$. Thus,

$$\begin{aligned} \text{nullity}(A) &= 5 - \text{rank}(A) = 5, \\ \text{nullity}(A') &= 5 - \text{rank}(A') = 4, \\ \text{nullity}(A'') &= 5 - \text{rank}(A'') = 3. \end{aligned}$$

\square