Math 314 Fall 2007 **Quiz #4** Name: Solutions

Show all of your work; answers without work may receive no credit. Make sure that you answer each question completely.

1. (10 pts) Find bases for row(A), col(A), and null(A).

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

Solution. We first find the reduced row echelon form R of A:

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 0 \end{bmatrix} \xrightarrow{\text{swap } R_1, R_3} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \end{bmatrix} \xrightarrow[R_3 - 2R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -3 & 3 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow{\text{swap } R_2, R_3} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & -3 & 3 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & -3 & 3 \end{bmatrix}$$

$$\xrightarrow[R_3 + 3R_2]{R_1 - 2R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} = R.$$

The nonzero rows $\left\{ \begin{bmatrix} 1 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \right\}$ of R form a basis for row(A).

Remember that col(R) $\neq$ col(A), but that the columns of R satisfy the same dependence relations as the columns of A. The columns of R with leading 1s are linearly independent and hence the corresponding columns of A are also linearly independent. Thus, $\left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \right\}$ is a basis for col(A).

The nullspace of A is the set of all solutions to $A\bar{x} = \bar{0}$. Using R, we see that the set of solutions is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_3 \\ 1x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}.$$

Hence, a basis for null(A) is $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$.

Note that rank(A) = dim(row(A)) = dim(col(A)) = 2 and nullity(A) = dim(null(A)) = 1, which sum to $n = 3$ as expected. □